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ANALYSIS OF COVARIANCE OF HEIGHT OF PINE STANDS

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Summary

The heights of trees from 24–, 33– and 35–year old stands of *Pinus sylvestris L* using co-variance analysis are compared, whereas the diameter of the breast height is the concomitant variable.

Key words and phrases: covariance, diameter of breast height, Scots pine (*Pinus sylvestris L*), height

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1. Introduction

One of the most important criteria of the productivity of the stand is the volume. The volume determining is based on the trees height and the diameters of the breast height (d.b.h.), i.e. the diameter measured at the height 1.3m, see Bruchwald (1973). In the paper we are interested in determining the relation between the height of the tree and d.b.h and in studying the properties of this relation. For our purpose we compare the height of trees and we eliminate the diameter influence using the method of covariance analysis. For the analysis we take 24–, 33–, 35–year old stands. The presented paper we treat as an introduction to research consisting in comparing relations between height of trees originating from different age groups and recognizable attributes having influence on the stand's height.

2. Experimental material

In the sample there are outcomes of 25 measurements of 24–year old trees from a sample plot area of 0.1ha, 25 measurements of 33–years old trees from a sample plot area of 0.15ha and 25 measurements of 35–years old trees from a sample plot area of 0.1ha. All trees grow in the Zielonka Experimental Forest District. The trees taken to the trial we choose using the Draudt method and they belong to the same age class of 20-40 years. In this class we compare mean heights of trees taking into consideration dependence of the height on d.b.h. For each tree we measure the arrow length and the diameter in two directions N-S and W-E taking the arithmetical mean from these two measurements as the real diameter.

3. Results

We use the statistical package SAS and Excel spreadsheet. The idea of the covariance analysis is to explain the response variable Y – height of trees by the confounding covariate with the regression analysis. In the preliminary analysis we study the assumptions of the covariance analysis:

1. The random variable Y depends on the variable X. Y is normally distributed and has homogeneous variances for age groups.
2. The concomitant variable X is random and has homogeneous variances for age groups, is normally distributed and has equal means for a particular age group.
3. There exists the relation between considered variables in each age group.
4. The slopes are the same for each age group.

For more details see Elandt (1964) and Oktaba (1972).

Using the Shapiro–Wilk test we check, if the heights of the stands (variable Y) and if the diameters (variable X) for particularly age groups are normally distributed, while Table 1 shows the calculations.

The results presented in the Table 1 imply that we do not have a basis for rejecting the hypothesis that the variable Y_i – height of i-year old stands is normally distributed for each i and we do not reject the hypothesis that the variable X_i – d.b.h. of - year old stands is normally distributed for each i, $i = 24, 33, 35$.

Table 1. Shapiro – Wilk test for the height and diameter

	Height		Diameter	
Age groups	Statistics W	*p*-value	Statistics W	*p*-value
24	0.950	0.253	0.983	0.933
33	0.972	0.692	0.944	0.186
35	0.961	0.426	0.973	0.730

Next, we test the hypothesis $H_0 : \sigma^2{}_{Y_{24}} = \sigma^2{}_{Y_{33}} = \sigma^2{}_{Y_{35}}$, where $\sigma^2{}_{Y_i}$ denotes the variance of the height for the i-year old stand with respect to the hypothesis that not all variances of heights of stands are the same. Based on the Bartlett test for 2 degrees of freedom the value of χ^2 is 0.184. The associated *p*-value is 0.912 and it implies, that we do not have a basis for rejecting the hypothesis that the variances of *Y* are homogenous.

In the successive research we study the null hypothesis $H_0 : \sigma^2{}_{X_{24}} = \sigma^2{}_{X_{33}} = \sigma^2{}_{X_{35}}$, where $\sigma^2{}_{X_i}$ denotes the variance of the diameter for the i-year old stand. Based on the Bartlett test for 2 degrees of freedom we have: the value χ^2 equals 2.664. The *p*-value 0.264 associated with variance of age merely shows that the variances are the same. It means that the concomitant variable *X* is random.

In order to verify, if the mean diameters are the same, we test the hypothesis $H_0 : \mu_{X_{24}} = \mu_{X_{33}} = \mu_{X_{35}}$, where μ_{X_i} denotes that the mean diameter for the stand at the age of i years, $i = 24, 33, 35$, with respect to the hypothesis: not all means of diameters are the same.

Table 2. Analysis of variance for diameter

Source	df	Sum of squares	Squares means	F statistics	*p*-value
Age groups	2	5.243	2.621	0.27	0.767
Errors	72	708.039	9.834		
Total	74	713.282			

Based on Table 2 we can conclude that the diameter is the random variable having equal means for particular age groups.

In order to determine the linear relation between the heights of the trees and the d.b.h. we fix the regression lines separately for each age group:

$$y_{24} = 0.335x + 9.078,\ y_{33} = 0.272x + 11.281,\ y_{35} = 0.308x + 9.702,$$

which indicate quantitatively how the mean height of the stands change with age and which you can see at Figure 1

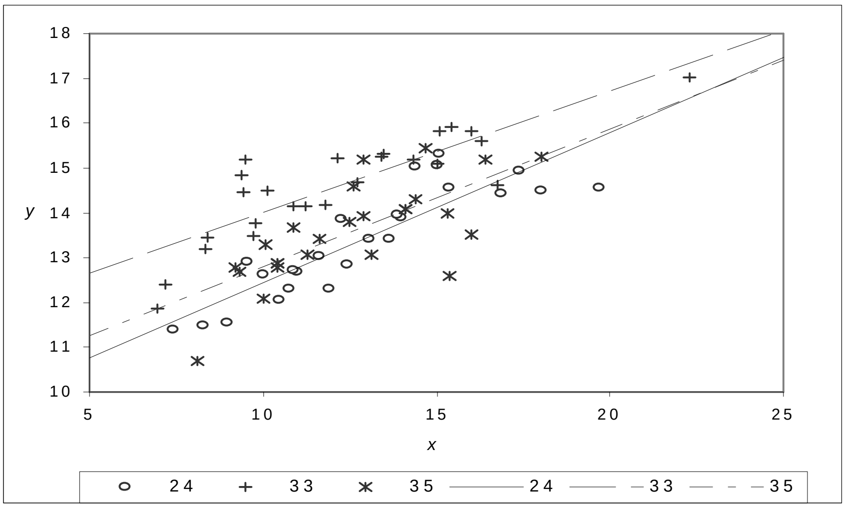

Fig. 1. Regression lines for 24, 33 and 35 – years old stands

Now, using the F test, we must check the relevance of these regressions, i.e. we test the null hypothesis $\beta_{1_i} = 0$ in rejecting $\beta_{1_i} \neq 0$ for $i = 24, 33, 35$, where β_{1_i} is the slope for the i age group. Based on Table 3 we can conclude that all regression coefficients β_{1_i} differ from zero.

Table 3. F test for critical regression coefficients

Age groups	F statistics	*p*-value
24	76.67	<0.0001
33	58.46	<0.0001
35	24.92	<0.0001

Some relations between the age of trees and their diameters are presented, see for example in Bruchwald (1986), Kaźmierczak, Grala–Michalak (2006).

For the presented covariance analysis it is important to show, that the regression lines are parallel. Hence, using the F test we check the hypothesis $H_0: \ \beta_{1_{24}} = \beta_{1_{33}} = \beta_{1_{35}}$. For 2 and 69 degrees of freedom we get the test value 0.61 and the associated *p*-value: 0.545. Hence, based on the research material, we can conclude, that the regression lines for 24–, 33– and 35– year old stands are parallel. The mean regression coefficient for three age groups is $b_w = 0.301$. The parallel regression lines for given age groups and for coefficient b_w are

$$y_{24} = 0.301x + 9.516, \ y_{33} = 0.301x + 10.923, \ y_{35} = 0.301x + 9.790,$$

where the remaining coefficients are determined by MIXED procedure in the package SAS, and the geometrical interpretation is presented at Figure 2.

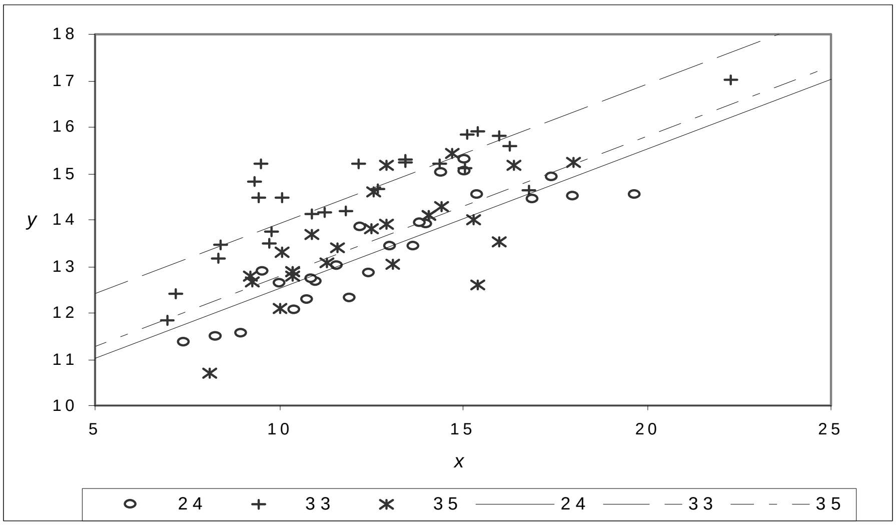

Fig. 2. Regression lines for 24–, 33– and 35 – year old stands (the same directions coefficients)

We test the hypothesis, the mean regression coefficient for age groups β_w equal 0 with respect to $\beta_w \neq 0$. For 1 and 71 degrees of freedom the F statistics is equal to 144.63 and *p*-value is smaller than 0.0001.

Thus the experimental material underlying all covariance assumptions is fulfilled or at least it does not deny.

Now, we consider the covariance analysis according to the model

$$y_{ij} = \mu + \tau_i + \beta(x_{ij} - \bar{x}_{..}) + \varepsilon_{ij},$$

where the observation y_{ij} is equal to the mean height μ and we add the effect of the i th age group τ_i and the regression coefficient for *Y* variable in regard to the variable *X* inside each age group β which is multiply by component $(x_{ij} - \bar{x}_{..})$, which represent participation of the d.b.h., the variable *X*. The sum is increased by random error ε_{ij}, $i = 24, 33, 35,$ $j = 1,2,...,25$.

In Table 4 we present the variance analysis of the height of the stand (*Y*) to rule out the diameter (*X*) influence.

Table 4. Variance analysis for the heights of the stands

Variation	df	Sum of squares	Squares mean	F statistics	*p*-value
Age group Error	2 72	21.310 95.564	10.610 1.327	8.06	0.0007
Total	74	116.84			

Based on the F test we can conclude that there are differences between age groups. The adjusted mean for the heights of the trees from 24 – year old stand is 13.295, for 33 – year old equals 14.702 and is 13.569 for the trees 35 – year old. Using the GLM Procedure for the covariance in the SAS and comparing age groups we can say, that groups of the 24 – year old and 35 – year old trees do not differ, while the 33 –year old trees are not the same. It could be generated by environment conditions, i.e. it is possible, that some trees are thickly planted. The analysis demands further investigating.

Table 5. The covariance analysis for the height of the stands

Variation	df	Regression deviation		F statistics	*p*-value
		$\sum(y-Y)^2$	The mean square		
The square sum for adjusted means	2	27.618	13.809	31.17	<0.0001
Inter age groups (Error)	71	31.467	0.443		
Total	73	59.085			

In Table 5 we present the covariance analysis for the height of the 24–, 33– and 35–year old stands. The statistics F is equal to 31.17 and *p*-value smaller then 0.0001 indicates that there is evidence of difference between the adjusted means for stands in tree age groups.

The relation between the height and the diameter is almost linear for each age group. Now, we try to determine one, common regression line and we test the hypothesis $H_0: \ \beta_w = \beta_g$ against the alternative, that not all coefficients are the same, where β_g there is the regression coefficient determined based on all measurements.

The statistics F is equal to 31.16 and for 2 and 71 degrees of freedom the *p*-value is smaller then 0.0001. Hence, we reject the null hypothesis and we are not able to determine one common regression line. Using the Tukey test we study the differences between the adjusted means for age groups, see Table 6.

Table 6. *p*-values for differences between age groups

	24	33
33	<0.0001	
35	0.320	<0.0001

The results given in Table 6 show, that there are significant differences between the adjusted means for 24–, 33– and 35–year old stands. The 24– and 35–year old stands are rather similar and they vary from the stand 33– year old.

4. Conclusions

Based on the presented analysis we can conclude, that for 24 –, 33 – and 35 – year old stands the relation between the height and the diameters exists and this relation is linear for each age group. Using the covariance analysis we also show, that the regression lines are parallel. After the elimination the diameter influence there are differences between the heights of 24–, 33– and 35–year old stands. The 33–year old stand differs from the other age groups. It could be caused by some atmosphere conditions or the neighborhood and it will be studied in the future.

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ANALIZA KOWARIANCJI WYSOKOŚCI DRZEW DLA DRZEWOSTANU SOSNOWEGO

Summary

W pracy porównujemy, wykorzystując analizę kowariancji, średnie wysokości drzew sosny zwyczajnej (*Pinus sylvestris L*) pochodzących z drzewostanów 24–, 33– i 35– letnich. Eliminujemy wpływ pierśnicy, którą traktujemy jako zmienną towarzyszącą.

Słowa kluczowe: kowariancja, pierśnica, sosna zwyczajna (*Pinus sylvestris* L), wysokość

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