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COMPARISON OF TWO ESTIMATION METHODS IN GROWTH CURVE MODEL WITH CONCOMITANT VARIABLES

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Summary

The method of parameters estimation in growth curve model with time moving concomitant variables considered in this paper is a peculiar case of the new two-stage method. Two methods: the first one given here and the second iterative method proposed by Wesołowska-Janczarek (1995) are compared using real data. Those data were connected with studying the fruit-bearing of different raspberry cultivars when the effect of the meteorological elements were eliminated.

Key words and phrases: methods of estimation, growth curve models, concomitant variables

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1. Introduction

The sum of profiles model was proposed by von Rosen (1985) and independently it was also proposed and used to the analysis of experimental data by Verbyla and Vanables (1988). Since then many problems connected with this kind of models have been considered by many authors. In some papers only two profiles are taken into consideration while in the others more of them. The general model sum of profiles is a form $Y = \sum_{i=1}^{n} A_i B_i T_i + E$ and it is regarded as the extended growth curve model. It is worth remembering that the growth curve model has been proposed by Potthoff and Roy (1964). In the model given by them there was only one profile and then one matrix of curves coefficients that have to be estimated. These curves are polynomials which describe changes of the studied feature in time.

Many of the papers concerned with sum of profiles models solve statistical problems in the case of different polynomial degree for various groups of units, then some matrices of parameters B_i are in the model. For example, it is present in von Rosen's paper (1989).

One of the groups of models with two profiles refers to the case when dependence between studied feature and concomitant variables moving in the time are taken into consideration. In this case, the first profile is connected with changing feature in the time and the second one defines relation between feature and concomitant variables. These kinds of models were considered by Wesołowska-Janczarek and Fus (1996), Wesołowska-Janczarek (1996a, 1996b), Wesołowska-Janczarek and others (1997).

In this group there can be distinguished some kinds of models depending on homogeneity of experimental units and on the fact whether reaction for all units on concomitant variables is the same or this reaction for separate groups is different. For example, the influence of meteorological elements on plants of various cultivars can be the same or different.

Then two profiles model can be considered as one of these forms:

$$Y = \mathbf{1}_n BT + \mathbf{1}_n \boldsymbol{\gamma}' X + E \tag{1.1}$$

when all units are homogeneous and the influence of concomitant variables for all units is the same,

$$Y = ABT + \mathbf{1}_n \boldsymbol{\gamma}' X + E \tag{1.2}$$

when units are divided into a groups and the influence is the same for all units and

$$Y = ABT + A\Gamma X + E \tag{1.3}$$

for a groups of units and when reaction to these groups on concomitant variables is different. Values of concomitant variables can be, in successive time points, the same for all units, for example in agricultural experiments where all plants are growing on field in the same conditions, like temperature or rainfall, or can be different as it is in economical or medical studies.

Maximum likelihood estimators of curve coefficients and covariance matrix for these models were given in papers by Wesołowska-Janczarek and Fus (1996) and Wesołowska-Janczarek (1996a). It is worth noticing that estimators in the model (1.2) are obtained by an iterative method which makes difficult to study their properties. Moreover, it is not possible to study a different influence of concomitant variables on the feature in various time points. To remove these difficulties a new two-stage method of estimation have been proposed. This method joins elements of two methods: two-stage seemingly unrelated regression (SUR) given by Zellner (1962) and common growth curve method given by Potthoff and Roy (1964). The suggestion of this new method, named hereafter two-stage method for model (1.2) was given by Wesołowska-Janczarek (2007). The aim of this paper is to show the application of this method in the case of model (1.2) and to compare it with the iterative method using real data.

2. SUR Method

Let $Y = [\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_p]$ be $n \times p$ matrix of observations of feature on n experimental units in p time points. The columns of Y denoted by $\mathbf{y}_j$ contain n measurements taken on n units in point of j $(j = 1, ..., p)$. Moreover, each of $\boldsymbol{y_j}$ is in regression relation with k_j concomitant (predeterminant) variables of x. This relation can be noted in the form

$$\mathbf{y}_j = X_j \boldsymbol{\beta}_j + \mathbf{u}_j; \qquad j = 1, ..., p, \tag{2.1}$$

where X_j is $n \times k_j$ matrix for each j the known value of concomitant variables in j time point for each of n elements, $\boldsymbol{\beta}_j$ is a vector of k_j unknown regression coefficients and $\mathbf{u}_j$ is $n \times 1$ vector of random errors.

In the first step of SUR estimators of $\boldsymbol{\beta}_j$ are obtained by least square method. They are as follows:

$$\hat{\boldsymbol{\beta}}_j = (X_j' X_j)^{-1} X_j' \mathbf{y}_j \quad \text{for} \quad j = 1, ..., p. \tag{2.2}$$

The covariance matrix of Y is of the form $\Sigma_Y = \Sigma_u \otimes I_n$ where $\Sigma_u = [\sigma_{jj'}]$, but if it is not known as the estimator of Σ_u can be used a matrix obtained using estimators of $\boldsymbol{\beta}_j$ given in (2.2). Then

$$\hat{\mathbf{u}}_j = \mathbf{y}_j - X_j \hat{\boldsymbol{\beta}}_j \tag{2.3}$$

and

$$\underset{pp}{\hat{\Sigma}_u} = [\hat{\mathbf{u}}'_j \hat{\mathbf{u}}_j] = [(\mathbf{y}_j - X_j \hat{\boldsymbol{\beta}}_j)'(\mathbf{y}_j - X_j \hat{\boldsymbol{\beta}}_j)]. \tag{2.4}$$

This estimator is used in the second step of SUR to correct all estimators of $\boldsymbol{\beta}_j$. Now, columns of Y are arranged into vector $\mathbf{y} = vec(Y)$ and system p equations (2.1) may be written as

$$\mathbf{y} = \begin{bmatrix} X_1 & 0 & \dots & 0 \\ 0 & X_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & X_p \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\beta}_1^* \\ \boldsymbol{\beta}_2^* \\ \dots \\ \boldsymbol{\beta}_p^* \end{bmatrix} + \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \dots \\ \mathbf{e}_p \end{bmatrix} = X\boldsymbol{\beta}^* + E. \tag{2.5}$$

Using this equation, estimators of $\boldsymbol{\beta}^*$ can be obtained by weighted least squares method.

$$\hat{\boldsymbol{\beta}}^* = (X'\Sigma^{-1}X)^{-1} X'\Sigma^{-1}\mathbf{y} \tag{2.6}$$

where Σ has to be replaced by the estimator given in (2.4).

Obtained in this way, corrected estimator $\hat{\boldsymbol{\beta}}^*$ is more efficient than $\hat{\boldsymbol{\beta}}_j$ (for $j = 1,\dots, p)$ given by (2.2). It is worth noticing that $\hat{\boldsymbol{\beta}}^*$ is the best unbiased linear estimator.

Estimators obtained in the first step and in the second one are the same when $\mathbf{u}'_j \mathbf{u}_{j'} = 0$ for all $j \neq j'$, or if all matrices X_j are equal, that is $X_1 = X_2 = \dots = X_p$. If y is normally distributed then $\hat{\boldsymbol{\beta}}^*$ is maximum likelihood estimator.

3. Iterative method

We consider now parameter estimation method in the growth curve model (1.2) with time moving concomitant variables, named hereafter the iterative method. Then model is in the following form

$$Y = ABT + \mathbf{1}_n \boldsymbol{\gamma}' X + E,$$

where Y is $n \times p$ matrix of observations, A is $n \times a$ known matrix which divides experimental units on a groups, B is $a \times q$ matrix of unknown coefficients in searched polynomials growth curves of $q-1$ degree, T is $q \times p$ matrix that include the successive powers of time points from 0 to $q-1$ (it is Vandermond's matrix), $\mathbf{1}_n$ is the vector of n ones, $\boldsymbol{\gamma}$ is a vector of s regression coefficients at concomitant variables, X is $s \times p$ matrix of values of these s variables in successive time points and E is a $n \times p$ matrix of random errors.

Estimators of parameters in this model obtained by maximum likelihood method under the normality assumptions $\underset{np}{Y} \sim N_{np}(ABT + \mathbf{1}_n \boldsymbol{\gamma}' X;\ \Sigma \otimes I_n)$ and $\Sigma > 0$ were given by Wesołowska-Janczarek and Fus (1996) in the following form:

$$\begin{aligned}
n\hat{\Sigma} &= (Y - A\hat{B}T - \mathbf{1}_n \hat{\boldsymbol{\gamma}}' X)'(Y - A\hat{B}T - \mathbf{1}_n \hat{\boldsymbol{\gamma}}' X) \\
\hat{B}_{\hat{\Sigma}} &= (A'A)^{-} A'(Y - \mathbf{1}_n \hat{\boldsymbol{\gamma}}' X)\hat{\Sigma}^{-1} T'(T\hat{\Sigma}^{-1} T')^{-1} \\
\hat{\boldsymbol{\gamma}}_{\hat{\Sigma}} &= [\mathbf{1}_n' Y - \mathbf{1}_n' A(A'A)^{-} A' Y \hat{\Sigma}^{-1} T'(T\hat{\Sigma}^{-1} T')^{-1} T]\hat{\Sigma}^{-1} X' R_{\hat{\Sigma}} \\
R_{\hat{\Sigma}} &= [nX\hat{\Sigma}^{-1} X' - \mathbf{1}_n' A(A'A)^{-} A' \mathbf{1}_n X\hat{\Sigma}^{-1} T'(T\hat{\Sigma}^{-1} T')^{-1} T\hat{\Sigma}^{-1} X']^{-1}.
\end{aligned} \tag{3.1}$$

It is necessary to start iteration from $\hat{\Sigma} = S = Y'[I_n - A(A'A)^{-} A]Y$ in order to calculate value of these estimators. The calculation is finished when none of the estimated elements of covariance matrix do not change more then given value ε, where ε is arbitrary small value. If $r(A) = a$ then general inverse of $A'A$ is common inverse matrix.

4. New two-stage method

Let $Y = [\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_p]$ be $n \times p$ matrix of observation as in previous parts of the paper, moreover, each of columns of this matrix is in regression relation with *s* other variables, that change values in time. For each of time points the relation has a form as in SUR method, but $k_1 = k_2 = ... = k_p = s$

$$\mathbf{y}_j = X_j \boldsymbol{\beta}_j + \mathbf{u}_j \text{ for } j = 1,2,..., p \tag{4.1}$$

then X_j is $n \times s$ matrix of *s* concomitant variables values in *j* time point, $\boldsymbol{\beta}_j$ is a vector of *s* unknown regression coefficients and $\mathbf{u}_j$ is a vector of random errors under assumptions $\varepsilon(\mathbf{u}_j) = \mathbf{0}$ and $\text{cov}(\mathbf{u}_j \mathbf{u}'_{j'}) = \sigma^2_{jj'} I_n$. The least squares estimators of $\boldsymbol{\beta}_j$ are obtained in the first step of this method

$$\hat{\boldsymbol{\beta}}_j = (X'_j X_j)^{-1} X'_j \mathbf{y}_j \quad \text{for } j = 1,...p. \tag{4.2}$$

Using these estimators we have then

$$\hat{\mathbf{u}}_j = \mathbf{y}_j - X_j \hat{\boldsymbol{\beta}}_j = [I_n - X_j (X'_j X_j)^{-1} X'_j] \mathbf{y}_j. \tag{4.3}$$

Estimated error vectors are next arranged into matrix $\hat{U} = [\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2, ..., \hat{\mathbf{u}}_p]$ that is used to appoint estimator of covariance matrix. This is $S = \hat{\Sigma} = \hat{U}'\hat{U}$ if all units are homogeneous or

$$S = \hat{U}'[I - A(A'A)^{-} A']\hat{U} \tag{4.4}$$

if units are divided into *a* groups. The partition is defined by matrix *A*.

In the second-step of this method the estimator of coefficients matrix in searched growth curves, according to Potthoff and Roy's method, will be obtained

$$\hat{B} = (A'A)^{-} A'Y\Sigma^{-1}T'(T\Sigma^{-1}T')^{-1} \tag{4.5}$$

where Σ is replaced by S given in (4.4).

If the values of s concomitant variables are the same for all units in each j time point, then matrices X_j will have the following form

$$X_j = \mathbf{1}_n \otimes [x_{j1}, x_{j2}, ..., x_{js}] \text{ for } j = 1,..., p \tag{4.6}$$

where $\otimes$ is Kronecker product of matrices. In example considered later in this paper matrices of concomitant variables values will have form as (4.6) because temperature, daily sum of precipitation and actual sunshine duration are the same for all plants on field.

If matrix X_j is like in (4.6) the following relation is true

$$\hat{U}'[I_n - A(A'A)^{-1}A']\hat{U} = Y'[I_n - A(A'A)^{-1}A']Y \tag{4.7}$$

that results from

$$\hat{U} = Y - [\mathbf{1}_n \otimes (\mathbf{1}_n'\mathbf{1}_n)^{-1}\mathbf{1}_n']Y = Y - \mathbf{1}_n \otimes [\bar{y}_1, \bar{y}_2, ..., \bar{y}_p]$$

and following matrix relations $A(B \otimes C) = AB \otimes C$ and $(B \otimes C)A = BA \otimes C$ where matrices A, B and vector C are of suitable dimensions.

Moreover, it is necessary to known, that if matrix X_j is of the form given in (4.6) and $s = 3$ the same as in our example then $X_j'X_j$ is following

$$X_j'X_j = n\begin{bmatrix} x_{j1}^2 & x_{j1}x_{j2} & x_{j1}x_{j3} \\ x_{j2}x_{j1} & x_{j2}^2 & x_{j2}x_{j3} \\ x_{j3}x_{j1} & x_{j3}x_{j2} & x_{j3}^2 \end{bmatrix} \tag{4.8}$$

and it is not of full rank because $\left|X_j'X_j\right| = 0$. Then it is not possible to obtain estimators of $\boldsymbol{\beta}_j$ as in (4.2) or to show regression relations between $\mathbf{y}_j$ and concomitant variables in each of time points. But it is possible to estimate regression relation taking into consideration all points together.

If we multiple left-hand of relation (4.1) by $(\mathbf{1}_n'\mathbf{1}_n)^{-1}\mathbf{1}_n$ and use (4.6), then $\bar{y}_j = [x_{j1}, x_{j2}, ..., x_{js}]\boldsymbol{\beta}_j + \bar{u}_j$ for $j = 1,..., p$. These means can be arranged into a vector

$$\begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \\ ... \\ \bar{y}_p \end{bmatrix} = \begin{bmatrix} [x_{11}, x_{12}, ..., x_{1s}]\boldsymbol{\beta}_1 \\ [x_{21}, x_{22}, ..., x_{2s}]\boldsymbol{\beta}_2 \\ ... \\ [x_{p1}, x_{p2}, ..., x_{ps}]\boldsymbol{\beta}_p \end{bmatrix} + \begin{bmatrix} \bar{u}_1 \\ \bar{u}_2 \\ ... \\ \bar{u}_p \end{bmatrix} \tag{4.9}$$

and if $\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = ... = \boldsymbol{\beta}_p = \boldsymbol{\gamma}$ and matrix of concomitant variable values is signified by X, the model will have form

$$\bar{\mathbf{y}} = X\boldsymbol{\gamma} + \bar{\mathbf{u}} \tag{4.10}$$

and the estimator of $\boldsymbol{\gamma}$ is following

$$\hat{\boldsymbol{\gamma}} = (X'X)^{-1}X'\bar{\mathbf{y}} \text{ where } \bar{\mathbf{y}} = Y'\mathbf{1}_n(\mathbf{1}_n'\mathbf{1}_n)^{-1}. \tag{4.11}$$

5. Example

Numerical data concern fruit-bearing of raspberry cultivars that is extended in time. Field researches were carried out by Experimental Station of Agricultural University in Felin near Lublin by workers of Orcharding Department.

In the experiment conducted in 1989, 16 cultivars of raspberry were compared with regard to yields, taking into consideration $p = 12$ time points. Values were taken from four plots for each of cultivars. In each of time points fruits were picked into 64 units. Three concomitant variables were taken into consideration: mean daily temperature from three days before harvest, daily sum of precipitation and actual sunshine duration.

The course of changes in fruit-bearing of these cultivars have been described by regression functions obtained by two methods: iterative and two-stage given in parts 3 and 4 of this paper.

Results of estimation obtained by these methods are following:

a) by iterative method $(\varepsilon = 1.0e^{-5}, 9 \text{ iteration})$ are:

$$\hat{B}_1 = \begin{bmatrix} 0.6334 & -0.1285 & 0.03749 & -0.002242 & 0.00003865 \\ 0.6090 & -0.1214 & 0.03775 & -0.002355 & 0.00004195 \\ 0.7158 & -0.2061 & 0.05562 & -0.003277 & 0.00005594 \\ 0.5950 & -0.1325 & 0.03607 & -0.002035 & 0.00003426 \\ 0.6535 & -0.1199 & 0.05754 & -0.003848 & 0.00007013 \\ 1.0348 & 0.1738 & -0.00151 & -0.000673 & 0.00001783 \\ 1.0204 & 0.0834 & 0.00426 & -0.000608 & 0.00001261 \\ 0.6906 & -0.2110 & 0.06503 & -0.004064 & 0.00007244 \\ 0.6280 & -0.1695 & 0.04257 & -0.002270 & 0.00003609 \\ 0.6313 & -0.1224 & 0.03917 & -0.002357 & 0.00004043 \\ 0.8907 & 0.4404 & -0.03454 & 0.000723 & -0.00000184 \\ 0.2860 & 0.2425 & 0.00971 & -0.001641 & 0.00003728 \\ 0.6323 & -0.1131 & 0.04607 & -0.002939 & 0.00005204 \\ 0.6772 & -0.2156 & 0.06253 & -0.003779 & 0.00006540 \\ 0.8146 & -0.3185 & 0.10260 & -0.006382 & 0.00011169 \\ 0.7582 & -0.1798 & 0.06937 & -0.004298 & 0.00007408 \end{bmatrix}$$

$$\hat{\Sigma}_1 = \begin{bmatrix} 0.01605 & 0.02142 & 0.00334 & 0.00464 & 0.01174 & 0.01412 & 0.01224 & -0.00465 & 0.01934 & 0.00566 & 0.00197 & -0.00042 \\ & 0.04392 & 0.00452 & 0.00546 & 0.02247 & 0.02566 & 0.02317 & 0.00248 & 0.03259 & 0.00427 & 0.00966 & 0.00416 \\ & & 0.06781 & 0.04161 & -0.01971 & -0.00191 & 0.03418 & -0.00779 & 0.02444 & -0.00949 & -0.02840 & 0.00066 \\ & & & 0.36309 & -0.00570 & -0.05131 & -0.01246 & 0.03325 & 0.01759 & -0.04644 & -0.01039 & -0.03323 \\ & & & & 0.06660 & 0.04262 & 0.01389 & 0.01697 & 0.03755 & 0.01834 & 0.01478 & 0.01610 \\ & & & & & 0.17082 & 0.09969 & 0.01726 & 0.04242 & 0.02019 & 0.01480 & 0.02257 \\ & & & & & & 0.26389 & 0.01961 & 0.06591 & 0.05881 & 0.00614 & 0.00125 \\ & & & & & & & 0.14976 & -0.00911 & 0.00953 & 0.03224 & 0.03392 \\ & & & & & & & & 0.14041 & 0.02172 & 0.00431 & 0.02546 \\ & & & & & & & & & 0.22878 & 0.01502 & 0.01113 \\ & & & & & & & & & & 0.07866 & 0.01191 \\ & & & & & & & & & & & 0.07484 \end{bmatrix}$$

$$\hat{\gamma}_1 = \begin{bmatrix} -0.01434 \\ 0.03013 \\ -0.08169 \end{bmatrix}, \qquad \left|\hat{\Sigma}_1\right| = 4.2617e-14$$

All curves obtained by this method are shown in the figure 1. In the figure 2 two curves are put for cultivars number 8 and 11.

b) by two-stage method

$$\hat{B}_2 = \begin{bmatrix} 0.0176 & 0.00758 & 0.00167 & -0.0000071 & -0.0000018 \\ -0.0068 & 0.01470 & 0.00193 & -0.000120 & 0.0000015 \\ 0.0999 & -0.07000 & 0.01980 & -0.00104 & 0.0000155 \\ -0.0209 & 0.00362 & 0.00025 & 0.00020 & -0.0000062 \\ 0.0377 & 0.01620 & 0.02170 & -0.00161 & 0.0000297 \\ 0.4190 & 0.31000 & -0.03730 & 0.00156 & -0.0000226 \\ 0.4050 & 0.22000 & 0.03160 & 0.00163 & -0.0000276 \\ 0.0747 & -0.07490 & 0.02920 & -0.00183 & 0.0000320 \\ 0.0122 & -0.03340 & 0.00675 & -0.00003 & -0.0000043 \\ 0.0154 & 0.01370 & 0.00335 & -0.00012 & 0.000000006 \\ 0.2750 & 0.57600 & -0.07040 & 0.00296 & -0.0000423 \\ -0.3300 & 0.37900 & -0.02610 & 0.00059 & -0.0000032 \\ 0.0164 & 0.02300 & 0.01030 & -0.00070 & 0.0000116 \\ 0.0613 & -0.07960 & 0.02670 & -0.00154 & 0.0000250 \\ 0.1990 & -0.18200 & 0.06680 & -0.00415 & 0.0000713 \\ 0.1420 & -0.04370 & 0.3360 & -0.00206 & 0.0000337 \end{bmatrix}$$

$$\hat{\Sigma}_2 = \begin{bmatrix} 0.8604 & 1.0280 & 0.3528 & 0.8869 & 0.5075 & 0.3150 & 0.2726 & -0.0379 & 0.7779 & 0.0585 & 0.0941 & -0.1175 \\ & 2.0334 & 0.6492 & 1.6047 & 0.9461 & 0.4061 & 0.5547 & 0.2843 & 1.2619 & -0.1705 & 0.4630 & -0.0246 \\ & & 2.3494 & 0.6714 & 0.1646 & 0.2803 & 0.6874 & -0.1352 & 0.7697 & 0.1171 & -0.6636 & 0.1674 \\ & & & 0.1187 & 1.8134 & 2.1035 & 0.4852 & 0.4442 & 2.2242 & -2.2132 & 0.6040 & -0.8789 \\ & & & & 2.9583 & 2.1110 & 1.6055 & 1.5033 & 2.2456 & 0.6029 & 0.1879 & 0.8738 \\ & & & & & 6.2672 & 3.6791 & 2.0959 & 1.4471 & -0.1821 & 0.7903 & 0.7119 \\ & & & & & & 0.1039 & 3.2578 & 2.0907 & 2.3383 & 1.1698 & 0.3838 \\ & & & & & & & 6.5866 & 1.0763 & 1.6435 & 1.6381 & 1.7832 \\ & & & & & & & & 6.5281 & 1.3334 & 0.9204 & 1.3949 \\ & & & & & & & & & 0.1168 & 0.9080 & 1.0029 \\ & & & & & & & & & & 4.2426 & 0.6951 \\ & & & & & & & & & & & 4.3571 \end{bmatrix}$$

$$\hat{\gamma}_2 = \begin{bmatrix} 0.00441 \\ 0.03054 \\ 0.06883 \end{bmatrix}, \qquad \left|\hat{\Sigma}_2\right| = 2.99e+06 .$$

Suitable curves are given in figures 3 and 4.

The values of generalized variance obtained in both of methods are given in semilogarithmic form.

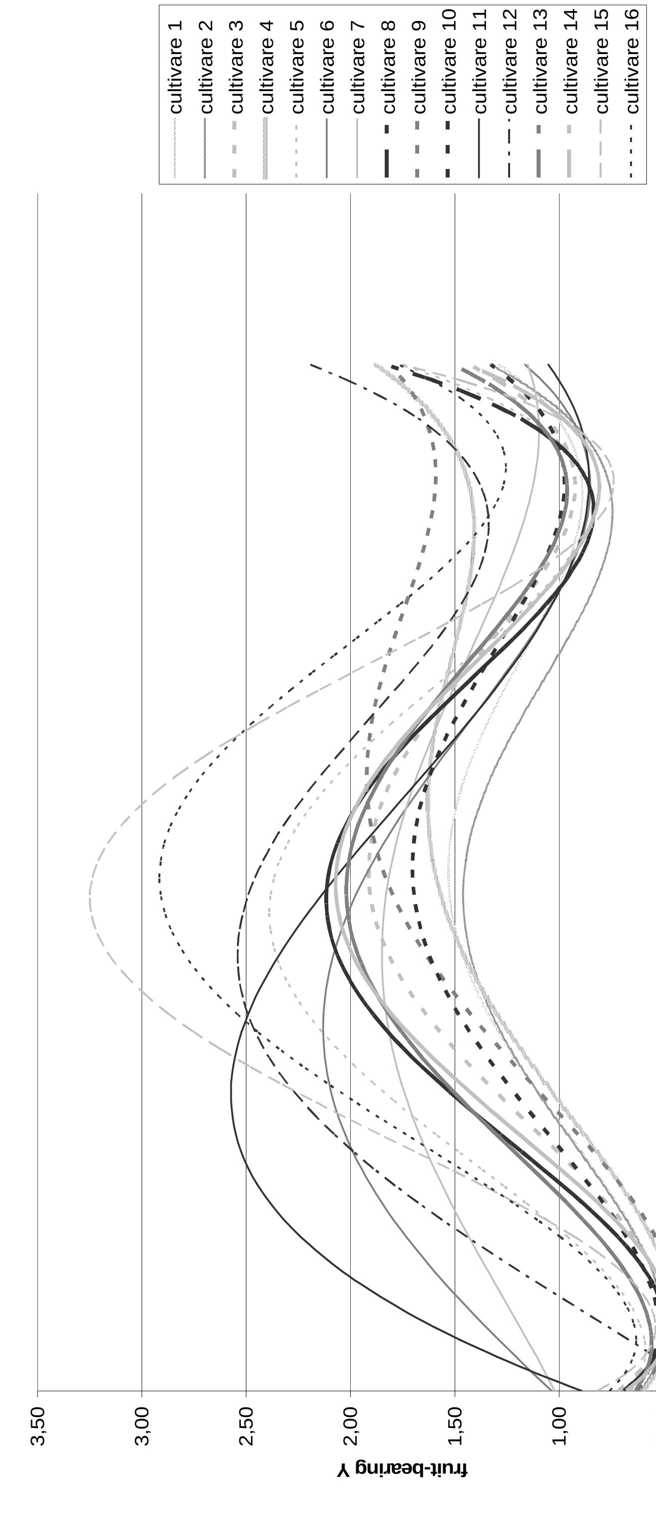

Fig. 1. Growth curves for all cultivars obtained by iterative method

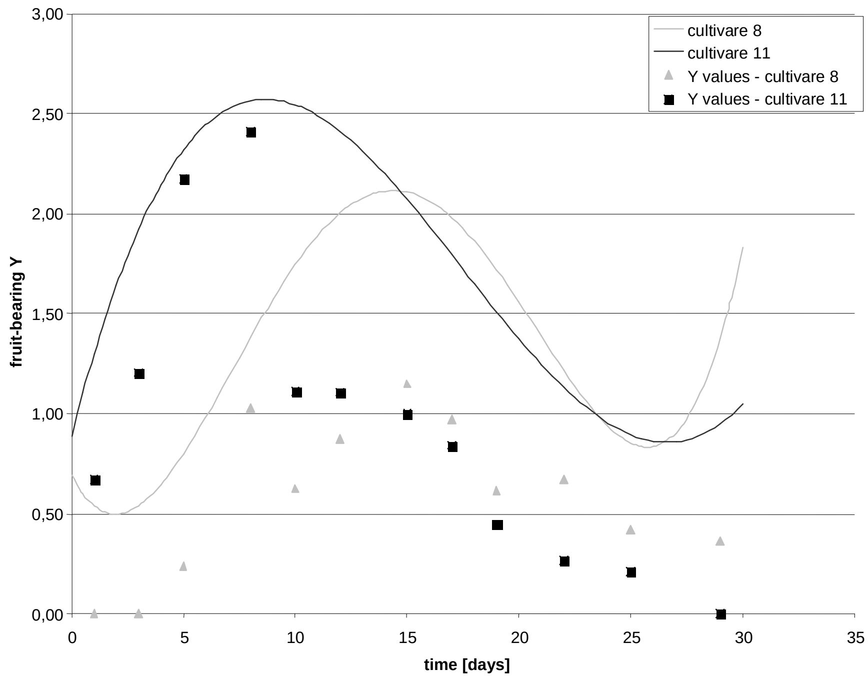

Fig. 2. Growth curves for 8th and 11th cultivar obtained by iterative method

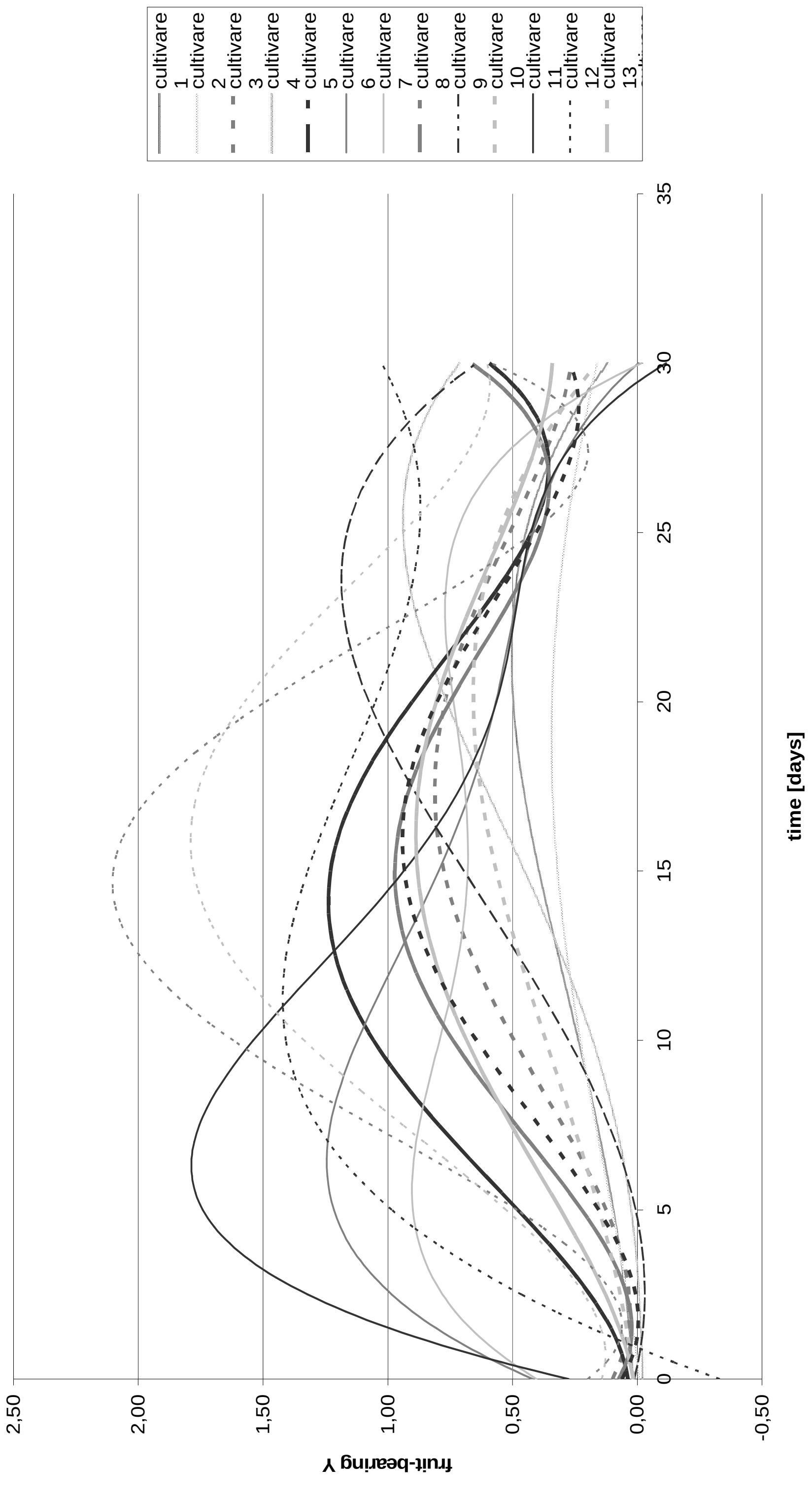

Fig 3. Growth curves for all cultivars obtained by two-stage method

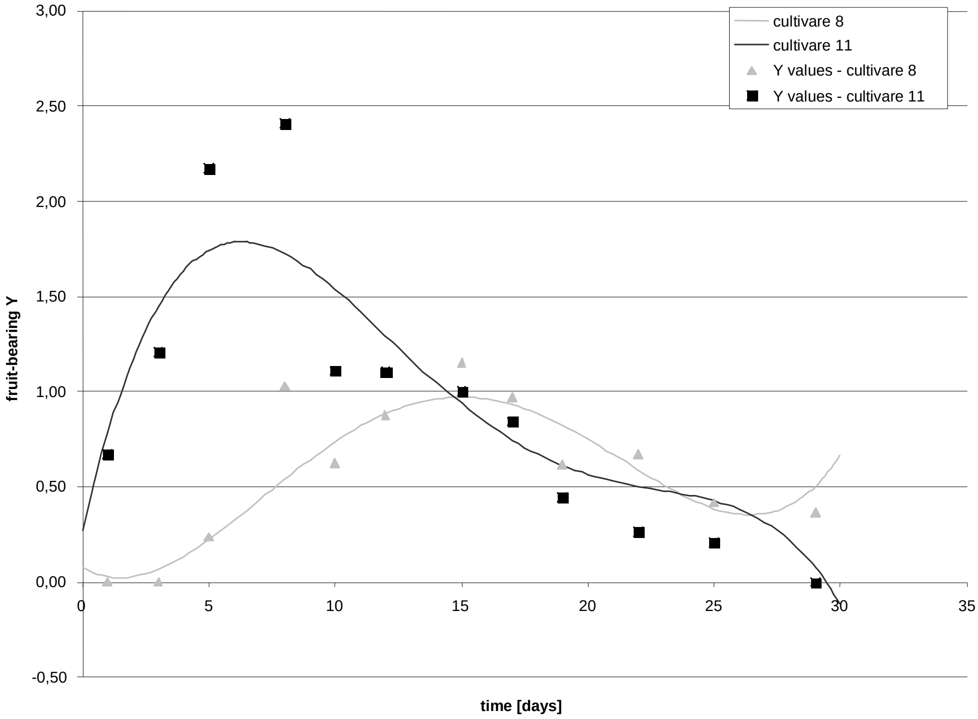

Fig 4. Growth curves for 8th and 11th cultivar obtained by two-stage method

6. Concluding remarks

It is known that research of estimators properties obtained by noniterative method is simpler than by iterative one but comparison of estimators obtained by presented methods in this paper require further study. The introductory conclusions of the comparison are following:

– the estimators of growth curves coefficients $\hat{B}_i \quad (i = 1,2)$ obtained by these methods are not the same,
– a new noniterative method differentiate curves for studied cultivars more (see figures 1 and 3 or 2 and 4),
– the generalized variance obtained by new method is greater than the same obtained by iterative method,
– in iterative method, beginning from second step of iteration, the elements of covariance matrix change very little,

– the estimators of $\boldsymbol{\gamma}_1$ and $\boldsymbol{\gamma}_2$ the vectors of regression coefficients of concomitant variance are very different with regard to their values as well as to their signs.

It is highly probable that the new method is better if each of matrices X_j are full rank - that is, if in each of time points values of concomitant variables for each of units are different. In this last case various regression reactions between studied feature and concomitant variables in each of time points can be estimated and additional hypothesis about equality of $\boldsymbol{\beta}_j \quad (j = 1,..., p)$ in (4.1) can be verified.

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PORÓWNANIE DWÓCH METOD ESTYMACJI W MODELU KRZYWYCH WZROSTU ZE ZMIENNYMI TOWARZYSZĄCYMI

Streszczenie

W niniejszej pracy przedstawiono metodę estymacji parametrów w modelu krzywych wzrostu z czasowo zmieniającymi się zmiennymi towarzyszącymi będącą szczególnym przypadkiem nowej metody dwustopniowej. Używając rzeczywistych danych porównano tę metodę z metodą iteracyjną zaproponowaną przez Wesołowską-Janczarek (1995). Dane doświadczalne dotyczyły badania przebiegu owocowania różnych odmian malin przy eliminacji wpływu warunków meteorologicznych.

Słowa kluczowe: metody estymacji, modele krzywych wzrostu, zmienne towarzyszące

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