Colloquium Biometricum 39
2009, 111–126

MODELLING AND ANALYSIS OF SPLIT-PLOT × SPLIT-BLOCK TYPE EXPERIMENTS WITH CONTROL C TREATMENTS

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Summary

The paper deals with a modelling and analysis of results of three factor experiments with control treatments which are included in levels of the third factor (*C* factor). It was assumed the experiments considered are carried out in incomplete split-plot × split-block designs. In particular a special attention was paid to possibilities and statistical consequences of applying the control treatments in the experiment. Moreover, tools are described which allow checking the general balance and stratum efficiency of the design, as well as the performance of the experiments in term of inference. Also a numerical example is presented to illustrate the method of the constructing the design considered and the analysis of data under mixed linear model.

Key words and phrases: augmented block design, control treatments, general balance, split-plot × split-block design, stratum efficiency, test treatments

Classification AMS 1993: 62K10, 62K15

1. Introduction

The purpose of this paper is to present a method of a designing three factor experiments with control treatments and a modelling data obtained from them. The experiments are laid out in split-plot × split-block (SPSB) design. The complete (orthogonal) SPSB designs are the most widely used in agriculture research (e.g. LeClerg et al., 1962, Mucha, 1975, Federer and King, 2007, Wadas et al., 2004, 2005). In field experiments certain treatments such as types of cultivation, application of irrigation water etc., may be necessary to arrange them in strips (rows or columns) across each block. Then it is convenient to arrange the plots of the design in the following way: the columns (or the rows) of the split-block design are split into smaller strips to accommodate the third factor. So, the third factor will be in the split-plot design in the relation to the column (or row) treatments.

In the present paper we will consider one of the cases of incomplete SPSB designs i.e. when a number of the levels of the third factor (say, *C*) is larger than the number of appropriate for them strips within each block. General methodology of the complete and incomplete SPSB designs, i.e. designing, modelling and statistical analysis, was presented in Ambroży and Mejza (2003, 2004*b*, 2006). Additionally in the present paper we assume that control treatments are included in the levels of the third factor. The case of the SPSB design, when some control treatments are connected with another factor was considered in Ambroży et al. (2008).

In the paper we are interested in the designs which have *general balance* property only.

General balance defines an important class of designs covering virtually all the traditional experimental designs and, in particular, those considered. In such designs all information matrices connected with treatment combinations mutually commute, and they have the same set of eigenvectors which define contrasts among treatment combination effects. It allows to joint information about the contrasts from different strata where they are estimated. With incomplete data sets, however, it may be difficult to fulfill all the conditions of general balance (Houtman and Speed, 1983, Mejza, 1992). In next chapter of the present paper we will remind the condition that should be fulfilled by the information matrices of treatment combinations of the SPSB designs.

In designing the considered three factor experiment we will use an augmented block design as generating subdesign, which statistical property can retain orthogonal block structure of the SPSB design, but allow unbalanced treatment structure. The incidence matrix of this subdesign is presented also in Kachlicka and Mejza (2000).

Augmented block designs (called also supplemented block designs) have been often used in planning factorial experiments, especially in a research with control or standard treatments (additional treatments). There are also situations in which an experimental material for certain treatments is limited. Then usually such treatments have less number of replications than the rest of treatments (see e.g. Caliński and Ceranka, 1974, Singh and Dey, 1979, Kachlicka et al., 2000, 2001)

In the modeling data obtained from such experiments we take into account the structure of an experimental material and a four-step randomization schema of the different kinds of units. With respect to the analysis of the obtained randomization model with seven strata we will adopt the approach, typical to the multistratum experiments with orthogonal block structure (cf. Nelder, 1965*a*, 1965*b*).

2. Assumptions and notations

Consider an $(s \times t \times w)$ – experiment in which the first factor, say A, has s levels $A_1, A_2, \ldots, A_s$, the second factor, say B, has t levels $B_1, B_2, \ldots, B_t$ and the third factor, say C, has w levels $C_1, C_2, \ldots, C_w$. Let v $(= stw)$ be the number of all treatment combinations.

We assume that a three factor experimental design structure is the following: we draw b blocks in such a way that they can be grouped into R superblocks, so each superblock contains b/R blocks. It should be underlined that number of superblocks and the number of blocks inside each superblock is strictly connected with an applied here constructing method of that design. Each block has a row-column structure with k_1 $(= s)$ rows and k_2 $(= t)$ columns of the first order, shortly, columns I. Then each column I has to be split into k_3 $(< w)$ columns of the second order, shortly, columns II. Here the rows correspond to the levels of the factor A, termed also as row treatments or A treatments, the columns I correspond to the levels of the factor B, called also column I treatments or B treatments, and the columns II are to accommodate the levels of the factor C termed as column II treatments or C treatments.

In the paper we consider the incomplete SPSB design with respect to the C treatments, additionally we assume the C treatments consist of two groups ($w = w_1 + w_2$) called test and control treatments, respectively.

Let us consider a randomization model of observations, the form and properties of which are strictly connected with the performed randomization processes in the experiment. The randomization scheme of the SPSB design consists of four randomization steps performed independently, that is by permuting blocks, rows, columns I and columns II. As a result the mixed model has the following form (Ambroży and Mejza, 2003, 2006)

$$\mathbf{y} = \boldsymbol{\Delta}'\boldsymbol{\tau} + \sum_{f=1}^{6} \mathbf{D}'_f \boldsymbol{\xi}_f + \mathbf{e}, \qquad E(y) = \boldsymbol{\Delta}'\boldsymbol{\tau}, \tag{2.1}$$

where $\mathbf{y}$ is n dimensional vector of lexicographically ordered observations, where $n = bstk_3$, $\boldsymbol{\Delta}'$ ($n \times v$) is a known design matrix for v treatment combinations, $\mathbf{D}'_1$ ($n \times b$), $\mathbf{D}'_2$ ($n \times bs$), $\mathbf{D}'_3$ ($n \times bt$), $\mathbf{D}'_4$ ($n \times btk_3$), $\mathbf{D}'_5$ ($n \times bst$), $\mathbf{D}'_6$ ($n \times n$), are design matrices for blocks, rows (within blocks), columns I (within blocks), column II (within columns I), whole plots (within blocks) and subplots (within whole plots) respectively, $\boldsymbol{\tau}$ ($v \times 1$) is the vector of fixed treatment combination effects, $\boldsymbol{\xi}_1$ ($b \times 1$), $\boldsymbol{\xi}_2$ ($bs \times 1$), $\boldsymbol{\xi}_3$ ($bt \times 1$), $\boldsymbol{\xi}_4$ ($btk_3 \times 1$), $\boldsymbol{\xi}_5$ ($bst \times 1$), $\boldsymbol{\xi}_6$ ($n \times 1$), $\mathbf{e}$ ($n \times 1$) are random effect matrices of blocks, rows, columns I, columns II, whole plots, subplots and technical errors, respectively.
The dispersion structure of the linear model (2.1) can be written as:

$$\mathbf{V}(\gamma) = \sum_{f=0}^{6} \gamma_f \mathbf{P}_f, \tag{2.2}$$

where γ_f are nonnegative variance components and the $\{\mathbf{P}_f\}$ are a family of known pairwise orthogonal projectors adding up to the identity matrix (cf. Houtman and Speed, 1983). The forms of these matrices are given in Ambroży and Mejza (2003, 2006). The range space $\Re\{\mathbf{P}_f\}$ of $\mathbf{P}_f$, f = 0, 1,…, 6, is termed the f-th stratum of the model and the $\{\gamma_f\}$ are unknown stratum variances. The ranks of the projectors $\mathbf{P}_f$ are as follows:

$$\mathrm{r}(\mathbf{P}_0)=1,\ \ \mathrm{r}(\mathbf{P}_1)=b-1,\ \ \mathrm{r}(\mathbf{P}_2)=b(\mathrm{s}-1),\ \ \mathrm{r}(\mathbf{P}_3)=b(t-1), \tag{2.3}$$

$$\mathrm{r}(\mathbf{P}_4)=bt(k_3-1),\ \ \mathrm{r}(\mathbf{P}_5)=b(s-1)(t-1),\ \ \mathrm{r}(\mathbf{P}_6)=b(s-1)\mathrm{t}(k_3-1)$$

From (2.2) and the properties of the projectors $\mathbf{P}_f$ it follows that considered design has an orthogonal block structure (cf. Nelder, 1965*a*, Houtman and Speed, 1983).

So, the model (2.1) can be analyzed using the methods developed for multistratum experiments. In this case, we have zero stratum (0) generated by the vector of ones, inter-block stratum (1), inter-row (within the block) stratum (2), inter-column I (within the block) stratum (3), inter-column II stratum (4) (within the column I), inter-whole plot (within the block) stratum (5), and inter-subplot (within the whole plot) stratum (6).

In this case we have 6 mentioned above main strata in which stratum analyses may be performed. The statistical analyses of submodels related to the different strata are based on algebraic properties of stratum information matrices for treatment combinations, which are defined as

$$\mathbf{A}_f=\Delta\mathbf{P}_f\Delta', \qquad f=1,2,\ldots,6. \tag{2.4}$$

The presented SPSB designs will be characterized according to their efficiency of an estimation of treatment combination comparisons (called also orthogonal (basic) contrasts) in the strata with respect to the following *general balance* property:

$$\mathbf{A}_f\mathbf{r}^{-\delta}\mathbf{A}_{f'}=\mathbf{A}_{f'}\mathbf{r}^{-\delta}\mathbf{A}_f \tag{2.5}$$

for $f,\ f'=1,2,\ldots,6$; $f\neq f'$ and $\mathbf{r}^{-\delta}=\mathrm{diag}(1/r_1,1/r_2,\ldots,1/r_v)$, where $\mathbf{r}$ is the vector of replications of the treatment combinations $\mathbf{r}=[r_1,r_2,\ldots,r_v]'$.

Stratum efficiency factors (noted by ε_{fh}) for a set of orthogonal contrasts (noted by $\mathbf{c}'_h\boldsymbol{\tau}$) are eigenvalues of the information matrices $\mathbf{A}_f$, $f=1,2,\ldots,6$ with respect to $\mathbf{r}^{-\delta}$. The contrasts are connected with comparisons among main effects of the considered factors and interaction effects between them.

In the considered SPSB designs the general hypotheses concerning factor *A* effects, factor *B* effects and the interaction $A \times B$ effects will be testable in one stratum only, which is appropriate for these effects. It can be shown (Ambroży and Mejza, 2006) that all contrasts among effects of the A treatments will be estimable in *the inter-row (within the block) stratum (2)*. It means that general hypothesis connected with the factor *A* is testable in this stratum only. Similarly, general hypothesis connected with the factor *B* will be testable in *the inter-column I (within the block) stratum (3)* only and general hypothesis connected with the interaction $A \times B$ will be testable in *the inter-whole plot (within the block) stratum (5)*. Other contrasts among main effects of the factor *C* and all interaction contrasts connected with this factor will be estimable in one stratum only or in two appropriate strata (see Example).

The necessary sum of squares for "treatments" (*f*) in Table 1 can be obtained from the formula

$$\mathrm{SST}_f = \sum_h \varepsilon_{fh} [(\widehat{\mathbf{c}'_h \boldsymbol{\tau}})_f]^2, \qquad f = 1, 2, ..., \quad 6$$

while the sum of squares for errors are as follows

$$\mathrm{SSE}_f = \mathrm{SSY}_f - \mathrm{SST}_f, \quad \text{where} \quad \mathrm{SSY}_f = \mathbf{y}'\mathbf{P}_f\mathbf{y}.$$

These sums are sufficient to build the appropriate *F*-tests.

Table 1. ANOVA in the *f*-th stratum, $f = 1, 2, .., 6$

Source of variation	DF	SS	E(MS)
"Treatments" (*f*)	$\nu_{Tf} = \mathrm{r}(\mathbf{A}_f)$	SST_f	$\nu_{Tf}\gamma_f + \boldsymbol{\tau}'\mathbf{A}_f\boldsymbol{\tau}$
Error (*f*)	$\nu_{Ef} = \nu_f - \nu_{Tf}$	SSE_f	$\nu_{Ef}\gamma_f$
Total (*f*)	$\nu_f = \mathrm{r}(\mathbf{P}_f)$	SSY_f	$\nu_f\gamma_f + \boldsymbol{\tau}'\mathbf{A}_f\boldsymbol{\tau}$

3. Construction method of the augmented SPSB designs

In the paper we consider one case of a construction of the augmented SPSB design using traditional method based on Kronecker product of matrices (⊗) (cf. Ambroży et al., 2004, Ambroży and Mejza, 2003, 2004*a*, 2006, Singh and Dey, 1979).

So the incidence matrix with respect to blocks of the SPSB design is of the form:

$$\mathbf{N}_1 = \mathbf{1}_s \otimes \mathbf{1}_t \otimes \mathbf{N}_C, \tag{3.1}$$

where $\mathbf{N}_C = \mathbf{N}_{d*}$ is an incidence matrix of the augmented subdesign d^* for C treatments (the column II treatments). The vectors $\mathbf{1}_s$ and $\mathbf{1}_t$ present one block incidence matrices for the factors A and B in the SPSB design. It means that A treatments and B treatments are in randomized complete block (RCB) subdesigns.

We assume the C treatments consist of two groups, test and control treatments, $w = w_1 + w_2$. The test C treatments (w_1) are allocated as in a RCB subdesign and additional (control) C (w_2) treatments as in an incomplete subdesign. So, the incidence matrix $\mathbf{N}_{d*}$ has the following form (see, Kachlicka and Mejza, 2000):

$$\mathbf{N}_{d*} = \begin{bmatrix} \mathbf{1}_{w_1}\mathbf{1}'_{b_3} \\ \mathbf{I}_R \otimes \mathbf{1}_q\mathbf{1}'_{b_3/R} \end{bmatrix}. \tag{3.2}$$

In this subdesign we have b_3 blocks with $\tilde{k}_3$ $(= w_1)$ units. The blocks are grouped into R superblocks of the same size (b_3/R blocks). The superblocks are then supplemented by q (different in each superblock) additional treatments, i.e. $w_2 = Rq$. So, the number of units inside each block in the design d^* is equal to $k_3 = \tilde{k}_3 + q$.

Let $\mathbf{C}_{d*}$ $(= \mathbf{C}_C)$ be information matrices for C treatments in the subdesigns. This matrix has two different eigenvalues $\varepsilon_0^* = 1$ and $\varepsilon_1^* = \dfrac{\tilde{k}_3}{k_3}$ with multiplicities equal to $\rho_0^* = 1 + R(q-1) + (w_1 - 1) = w_1 + w_2 - R$ and $\rho_1^* = R - 1$, respectively. It can be shown that the first class of efficiency equal to ε_0^* (= 1) is connected with the comparison 1) between the test group and the control group of the C treatments (C^T *vs.* C^C), 2) among control C treatments inside each superblock (C_1^C), 3) the test C treatments only (C^T). The second class of efficiency equal to ε_1^* refers to the comparisons among the control C treatments between the superblocks (C_2^C).

Finally, parameters of the incomplete SPSB design are as follows

$$v = stw\,,\ b = b_3\,,\ k = stk_3\,,\ \mathbf{r} = \mathbf{1}_s \otimes \mathbf{1}_t \otimes \mathbf{r}_C\,,\ \mathbf{r}_C = [b_3\mathbf{1}'_{w_1} \vdots (b_3/R)\mathbf{1}'_{w_2}]' \quad (3.3)$$

where v, b, k, $\mathbf{r}$, $\mathbf{r}_C$ denote the number of the treatment combinations, the number of the blocks, the size of the blocks, the vector of replication of the treatment combinations, the vector of replication of C treatments, respectively.

4. Example

To illustrate the theory presented in the paper, consider a (2 × 2 × 7) – experiment of type SPSB. Note the number of A treatments and the number of B treatments are equal to two, so $s = t = 2$ while the number of C treatments $w = 7$. Suppose that the C treatments are allocated in the columns II according to the incidence matrix given in (3.2):

$$\mathbf{N}_{d*} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

It was assumed (only for illustration) that three test C treatments ($w_1 = 3$) are allocated in a RCB design in four blocks ($b_3 = 4$) of size equal to 3 ($\tilde{k}_3 = 3$). These blocks are grouped into two ($R = 2$) superblocks, each composed of two blocks. Each superblock (with two blocks) of the RCB design is augmented with q = 2 different control C treatments. So, in the experiment $w_2 = Rq = 4$ different control C treatments will appear. The parameters of the augmented subdesign for the C treatments are as follows:

$$w = 7,\ b_3 = 4,\ k_3 = 5,\ \mathbf{r}_C = [4, 4, 4, 2, 2, 2, 2]',\ \varepsilon_0^* = 1,\ \rho_0^* = 5,\ \varepsilon_1^* = 0{,}6,\ \rho_1^* = 1.$$

Remaining factors, A and B ($s = t = 2$) are as in a complete (orthogonal) SPSB design.

Finally, the considered SPSB design is described by the incidence matrix $\mathbf{N}_1$ given in (3.1) and has the form $\mathbf{N}_1 = \mathbf{1}_2 \otimes \mathbf{1}_2 \otimes \mathbf{N}_{d*}$. In accordance to (3.3) its parameters are:

$$s = t = 2,\ w = 7,\ v = stw = 28,\ b = 4,\ k = 20,\ \mathbf{r} = \mathbf{1}_2 \otimes \mathbf{1}_2 \otimes [4, 4, 4, 2, 2, 2, 2]'.$$

Figure 1 shows a row-column structure of one block in the SPSB design and its division into smaller strips.

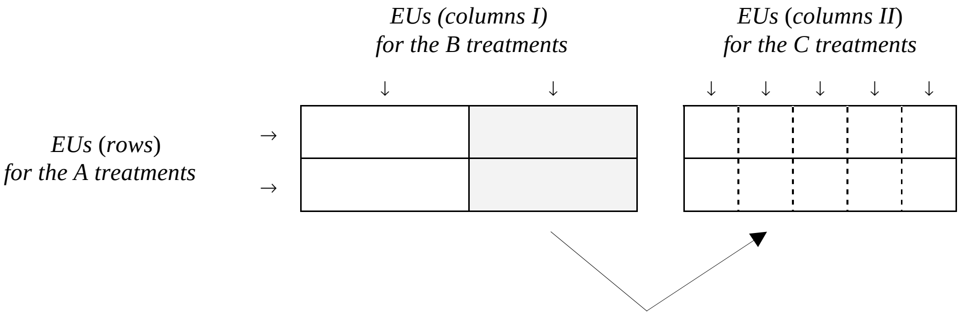

Fig. 1. The structure of experimental units of a different order inside each block in the considered SPSB design

The sample layout (before four step randomization) of the augmented SPSB design in the Example is illustrated with Figure 2.

Superblock 1

Block 1

	B_1					B_2				
	C_1	C_2	C_3	C_4	C_5	C_1	C_2	C_3	C_4	C_5
A_1										
A_2										

Block 2

	B_1					B_2				
	C_1	C_2	C_3	C_4	C_5	C_1	C_2	C_3	C_4	C_5
A_1										
A_2										

Superblock 2

Block 3

	B_1					B_2				
	C_1	C_2	C_3	C_6	C_7	C_1	C_2	C_3	C_6	C_7
A_1										
A_2										

Block 4

	B_1					B_2				
	C_1	C_2	C_3	C_6	C_7	C_1	C_2	C_3	C_6	C_7
A_1										
A_2										

Fig. 2. The sample layout (before randomization process) of the considered SPSB experiment design

According to the above plan the A treatments, B treatments and C treatments are allocated on the rows, columns I and columns II, respectively. The treatment combinations with the control treatments $C_4 - C_7$ are replicated twice in this experiment.

Statistical properties of the considered design are strictly connected with the algebraic properties of the stratum information matrices for the treatment combinations (2.4). These matrices are (cf. Ambroży and Mejza, 2006):

$$\mathbf{A}_1 = \frac{1}{20}\mathbf{J}_4 \otimes (\mathbf{N}_{d*}\mathbf{N}'_{d*} - \frac{1}{4}\mathbf{r}_C\mathbf{r}'_C), \qquad \mathbf{A}_3 = \frac{1}{10}\mathbf{J}_2 \otimes (\mathbf{I}_2 - \frac{1}{2}\mathbf{J}_2) \otimes \mathbf{N}_{d*}\mathbf{N}'_{d*},$$

$$\mathbf{A}_2 = \frac{1}{10}(\mathbf{I}_2 - \frac{1}{2}\mathbf{J}_2) \otimes \mathbf{J}_2 \otimes \mathbf{N}_{d*}\mathbf{N}'_{d*}, \qquad \mathbf{A}_4 = \frac{1}{2}\mathbf{J}_2 \otimes \mathbf{I}_2 \otimes (\mathbf{r}_C^{\delta} - \frac{1}{5}\mathbf{N}_{d*}\mathbf{N}'_{d*}),$$

$$\mathbf{A}_5 = (\mathbf{I}_2 - \frac{1}{2}\mathbf{J}_2) \otimes (\mathbf{I}_2 - \frac{1}{2}\mathbf{J}_2) \otimes \frac{1}{5}\mathbf{N}_{d*}\mathbf{N}'_{d*}, \qquad \mathbf{A}_6 = (\mathbf{I}_2 - \frac{1}{2}\mathbf{J}_2) \otimes \mathbf{I}_2 \otimes (\mathbf{r}_C^{\delta} - \frac{1}{5}\mathbf{N}_{d*}\mathbf{N}'_{d*}),$$

where

$$\mathbf{N}_{d*}\mathbf{N}'_{d*} = \begin{bmatrix} 4 & 4 & 4 & 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 0 & 0 \\ 2 & 2 & 2 & 2 & 2 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 & 2 & 2 \\ 2 & 2 & 2 & 0 & 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 4\mathbf{1}_3\mathbf{1}'_3 & 2\mathbf{1}_3\mathbf{1}'_4 \\ 2\mathbf{1}_4\mathbf{1}'_3 & \mathbf{I}_2 \otimes 2\mathbf{1}_2\mathbf{1}'_2 \end{bmatrix},$$

$$\mathbf{r}_C\mathbf{r}'_C = \begin{bmatrix} 16 & 16 & 16 & 8 & 8 & 8 & 8 \\ 16 & 16 & 16 & 8 & 8 & 8 & 8 \\ 16 & 16 & 16 & 8 & 8 & 8 & 8 \\ 8 & 8 & 8 & 4 & 4 & 4 & 4 \\ 8 & 8 & 8 & 4 & 4 & 4 & 4 \\ 8 & 8 & 8 & 4 & 4 & 4 & 4 \\ 8 & 8 & 8 & 4 & 4 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 16\mathbf{1}_3\mathbf{1}'_3 & 8\mathbf{1}'_3\mathbf{1}'_4 \\ 8\mathbf{1}'_4\mathbf{1}'_3 & 4\mathbf{1}'_4\mathbf{1}'_4 \end{bmatrix}.$$

It can be shown that above information matrices satisfy the condition (2.5).

The eingenvalues of these information matrices calculated with respect to $\mathbf{r}^{\delta}$ are called stratum efficiency factors (see Table 2). They refer to the eigen-vectors which generate orthogonal contrasts.

Let

$\mathbf{a}_1 = \mathbf{b}_1 = [1, -1]'/\sqrt{2}$, $\mathbf{a}_2 = \mathbf{b}_2 = [1, 1]'/\sqrt{2}$,

$\mathbf{c}_1 = [1, -1, 0, 0, 0, 0, 0]'/2\sqrt{2}$, $\mathbf{c}_5 = [0, 0, 0, 1, 1, -1, -1]'/2\sqrt{2}$,

$\mathbf{c}_2 = [1, 1, -2, 0, 0, 0, 0]'/2\sqrt{6}$, $\mathbf{c}_6 = [4, 4, 4, -3, -3, -3, -3]'/2\sqrt{66}$,

$\mathbf{c}_3 = [0, 0, 0, 1, -1, 0, 0]'/2$, $\mathbf{c}_7 = [1, 1, 1, 1, 1, 1, 1]'/\sqrt{20}$.

$\mathbf{c}_4 = [0, 0, 0, 0, 0, 1, -1]'/2$,

.

The eigenvectors $\mathbf{p}_h = \mathbf{a}_j \otimes \mathbf{b}_k \otimes \mathbf{c}_l$ (h = 1, 2,..., 28; $j = k$ = 1, 2; l = 1, 2,..., 7) are $\mathbf{r}^{\delta}$-orthonormal, i.e., satisfy the conditions $\mathbf{p}'_h \mathbf{r}^{\delta} \mathbf{p}_h = 1$ and $\mathbf{p}'_h \mathbf{r}^{\delta} \mathbf{p}_{h'} = 0$, for $h \neq h'$, h, h' = 1, 2,..., 28. Since $\mathbf{A}_f \mathbf{1}_{28} = \mathbf{0}$, $f > 0$, the last eigenvector $\mathbf{p}_{28}$ may be chosen as $\frac{1}{\sqrt{n}} \mathbf{1}_{28}$, where n = 80. Let us note that $\mathbf{c}_h = \mathbf{r}^{\delta} \mathbf{p}_h$ (h < 28) define (basic) contrasts of the form $\mathbf{c}'_h \boldsymbol{\tau}$, h = 1, 2, ..., 27. They play fundamental role in an investigation of statistical properties of the SPSB design, in ANOVA and in a statistical inference.

Statistical properties which are necessary in ANOVA of the augmented SPSB design are given in Table 2. All calculations can be do by different programs for example Excel and GenStat. Degrees of freedom connected with the contrasts stand for the numbers of estimable contrasts in each stratum. The ranks of the projectors $\mathbf{P}_f$ were obtained from (2.3).

It can be noticed that using the augmented SPSB experiment design from the Example we loss information about the contrasts among the control C treatments (C_2^C) and interaction contrasts connected with them, only. These contrasts are estimated with not full efficiency in two strata. The remaining contrasts are estimable with full efficiency in appropriate stratum as in a complete SPSB design (A, B, A × B, C^T, C_1^C, C^T vs. C^C, A × C^T, A × C_1^C, A × (C^T vs. C^C), B × C^T, B × C_1^C, B × (C^T vs. C^C), A × B × C^T, A × B × C_1^C, A × B × (C^T vs. C^C)).

Table 2. Stratum efficiency factors corresponding to estimable orthogonal contrasts from the Example

Sources of variation	Degrees of freedom	Efficiency factors
	the inter-block stratum (1)	
control C treatments C_2^C	$\rho_1^* = 1$	$1 - \varepsilon_1^* = 0,4$
Error (1)	$r(\mathbf{P}_1) - 1 = 2$	
	the inter-row (within the block) stratum (2)	
A	$s - 1 = 1$	1
$A \times C_2^C$	$(s-1)\rho_1^* = 1$	$1 - \varepsilon_1^* = 0,4$
Error (2)	$r(\mathbf{P}_2) - 2 = 2$	
	the inter-column I (within the block) stratum (3)	
B	$t - 1 = 1$	1
$B \times C_2^C$	$(t-1)\rho_1^* = 1$	$1 - \varepsilon_1^* = 0,4$
Error (3)	$r(\mathbf{P}_3) - 2 = 2$	

	the inter-column II (within the column I) stratum (4)	
C	$w - 1 = 6$	
test C treatments C^T	$\rho_0^* - 3 = 2$	1
control C treatments C_1^C	$\rho_0^* - 3 = 2$	1
control C treatments C_2^C	$\rho_1^* = 1$	$\varepsilon_1^* = 0{,}6$
C^T *vs.* C^C	1	1
$B \times C$	$(t - 1)(w - 1) = 6$	
$B \times C^T$	$(t-1)(\rho_0^* - 3) = 2$	1
$B \times C_1^C$	$(t-1)(\rho_0^* - 3) = 2$	1
$B \times C_2^C$	$(t-1)\rho_1^* = 1$	$\varepsilon_1^* = 0{,}6$
$B \times (C^T$ *vs.* $C^C)$	$(t-1) \cdot 1 = 1$	1
Error (4)	$r(\mathbf{P}_4) - 12 = 20$	
	the inter-whole plot (within the block) stratum (5)	
$A \times B$	$(s - 1)(t - 1) = 1$	1
$A \times B \times C_2^C$	$(s-1)(t-1)\rho_1^* = 1$	$1 - \varepsilon_1^* = 0{,}4$
Error (5)	$r(\mathbf{P}_5) - 2 = 2$	
	the inter-subplot (within the whole plot) stratum (6)	
$A \times C$	$(s - 1)(w - 1) = 6$	
$A \times C^T$	$(s-1)(\rho_0^* - 3) = 2$	1
$A \times C_1^C$	$(s-1)(\rho_0^* - 3) = 2$	1
$A \times C_2^C$	$(s-1)\rho_1^* = 1$	$\varepsilon_1^* = 0{,}6$
$A \times (C^T$ *vs.* $C^C)$	$(s-1) \cdot 1 = 1$	1
$A \times B \times C$	$(s - 1)(t - 1)(w - 1) = 6$	
$A \times B \times C^T$	$(s-1)(t-1)(\rho_0^* - 3) = 2$	1
$A \times B \times C_1^C$	$(s-1)(t-1)(\rho_0^* - 3) = 2$	1
$A \times B \times C_2^C$	$(s-1)(t-1)\rho_1^* = 1$	$\varepsilon_1^* = 0{,}6$
$A \times B \times (C^T$ *vs.* $C^C)$	$(s-1)(t-1) \cdot 1 = 1$	1
Error (6)	$r(\mathbf{P}_6) - 12 = 20$	

4. Remarks

1. Further statistical analysis connected with general and particular hypotheses can be performed according to procedures given in Ambroży and Mejza (2006).

2. Statistical inferences (estimates and tests) about the contrasts which are estimated in two strata can be obtained using the information separately from one stratum only or performing for them the combined estimation and testing based on information from these strata in which they are estimable. The combined estimators usually possesse better statistical properties than the stratum BLUEs of the same contrast. Hence, the estimator combining is worth considering (Caliński and Kageyama, 2000). Some combining methods of information from two strata are described in Ambroży and Mejza (2006) also.

References

Ambroży K., Kachlicka D., Mejza I. (2004). Rozszerzone układy blokowe w konstrukcji układów mieszanych dla doświadczeń trójczynnikowych. *Colloquium Biometryczne* 34, 15–26 (in Polish).

Ambroży K., Kachlicka D., Mejza I. (2008). Split-plot x Split-block analysis with control A treatments. *Colloquium Biometricum* 38, 173–183.

Ambroży K., Mejza I. (2003). Some split-plot × split-block designs. *Colloquium Biometryczne* 33, 83–96.

Ambroży K., Mejza I. (2004*a*). Incomplete split-plot × split-block designs based on Kronecker type products. *Colloquium Biometryczne* 34, 27–38.

Ambroży K., Mejza I. (2004*b*). Split-plot × split-block type three factors designs. *Proc. of the 19th International Workshop on Statistical Modelling*, Florence, 291–295.

Ambroży K., Mejza I. (2006). *Doświadczenia trójczynnikowe z krzyżową i zagnieżdżoną strukturą poziomów czynników*. Wyd. Polskie Towarzystwo Biometryczne and PRODRUK, Poznań (in Polish).

Caliński T., Ceranka B. (1974). Supplemented block designs. *Biom. J*. 16, 299–305.

Caliński T., Kageyama S. (2000). *Block Designs: A Randomization Approach, Volume I: Analysis. Lecture Notes in Statistics* 150, Springer-Verlag, New York.

Federer W. T., King F. (2007). *Variations on Split Plot and Split Block Experiment Designs*. Wiley. New Jersey.

Houtman A.M., Speed T.P. (1983). Balance in designed experiments with orthogonal block structure. *Ann. Statist*. 11, 1069–1085.

Kachlicka D., Mejza I. (2000). Rozszerzone α-rozkładalne układy blokowe. *Bibliotheca Fragmenta Agronomica*, T. 7, 59–76.

Kachlicka D., Mejza I., Jokś W., Zwierzykowski Z. (2000). Statistical analysis of a preliminary experiment with new Festulolium cultivars laid out in an incomplete block design. *Sbornik referátov XIV. letni školy biometriky*, Słowacja, 103–109.

Kachlicka D., Kaczmarek Z., Mejza I., Jokś W., Zwierzykowski Z. (2001). Multivariate methods used in analysis of four-cut trial on Festulolium cultivars. *Colloquium Biometryczne* 31, 113–121.

LeClerg E.L., Leonard W.H., Clark A.G. (1962). *Field plot technique*. Burgess, Minneapolis.

Mejza S. (1992). On some aspects of general balance in designed experiments. *Statistica* 52, 263–278.

Mucha S. (1975). Reakcja odmian pszenicy jarej i ozimej na Antywylegacz. *Wiadomości Odmianoznawcze*, Rok II, Zeszyt 2/3, COBORU, Słupia Wielka.

Nelder, J. A. (1965*a*). The analysis of randomized experiments with orthogonal block structure. 1. Block structure and the null analysis of variance. *Proc. of the Royal Soc. of Lond. Ser. A*, 283, 147–162.

Nelder, J. A. (1965*b*). The analysis of randomized experiments with orthogonal block structure. 2. Treatment structure and general analysis of variance. *Proc. of the Royal Soc. of Lond. Ser. A*, 283, 163–178.

Singh M., Dey A. (1979). On analysis of some augmented block designs. *Biom. J*. 21, 87–92.

Wadas W., Jabłońska-Ceglarek R., Kosterna E. (2004). The effect of the cultivation method and nitrogen fertilization on the size and structure of the field of immature potato tubers. *Electronic Journal of Polish Agricultural Universities. Horticulture*, 7 (1), art-07.html.

Wadas W., Jabłońska-Ceglarek R., Kosterna E. (2005). The nitrates content in early potato tubers depending on growing conditions. *Electronic Journal of Polish Agricultural Universities. Horticulture*, 8 (1), art-26.html.

MODELOWANIE I ANALIZA DOŚWIADCZEŃ TYPU SPLIT-PLOT × SPLIT-BLOCK Z OBIEKTAMI KONTROLNYMI W OBRĘBIE CZYNNIKA C

Streszczenie

Praca dotyczy modelowania i analizy wyników trójczynnikowych doświadczeń z obiektami kontrolnymi, które są włączone do poziomów trzeciego czynnika (C). Przyjęto, że doświadczenia były założone w układzie niekompletnym split-plot × split-block. Szczególną uwagę zwrócono w pracy na możliwości i konsekwencje zastosowania obiektów kontrolnych w doświadczeniu. Ponadto, zostały opisane narzędzia pozwalające na sprawdzenie zarówno właściwości ogólnego zrównoważenia i warstwowej efektywności układu, jak i możliwości wnioskowania z tego typu doświadczeń. Przedstawiono także numeryczny przykład, ilustrujący metodę konstrukcji rozważanego układu i analizy danych przy modelu liniowym mieszanym.

Słowa kluczowe: rozszerzony układ blokowy, obiekty kontrolne, ogólne zrównoważenie, układ split-plot × split-block, warstwowa efektywność, obiekty testowe

Klasyfikacja AMS 2000: 62K10, 62K15