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TABLES FOR SHAPIRO–WILK W STATISTIC ACCORDING TO ROYSTON APPROXIMATION

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Dedicated to the memory of Professor Wiktor Oktaba.

Summary

Tables of coefficients and critical values for Shapiro–Wilk test of normality, calculated according to approximation given by Royston (1992), for $n = 4(1)58$ and significance levels $\alpha = 0.01, 0.02, 0.05, 0.1$ are enclosed. It is shown that original tables by Shapiro and Wilk in 1965 give type I error a little beyond the nominal significance level.

Keywords and phrases: Shapiro–Wilk *W* statistic, test for normality, Type I error

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1. Introduction

The Shapiro–Wilk W statistic (1965) of the form

$$W = \frac{\left[\sum_{i=1}^{n} a_i x_{(i)}\right]^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \tag{1.1}$$

is considered by many authors as the best statistic for checking univariate normality of data, especially for small sample sizes. Small values of the statistic W indicate nonnormality. In the formula (1.1) $x_{(i)}$ are ordered values of the sample $x_1, x_2, \ldots, x_n$, i.e. $x_{(1)} \le x_{(2)} \le \ldots \le x_{(n)}$. The exact values of coefficients a_i are expressed as follows

$$\mathbf{a} = [a_1, a_2, \ldots, a_n]' = \mathbf{m}'\mathbf{V}^{-1}\left[\mathbf{m}'\mathbf{V}^{-1}\mathbf{V}^{-1}\mathbf{m}\right]^{-\frac{1}{2}},$$

where $\mathbf{m} = E\left[x_{(1)}, x_{(2)}, \ldots, x_{(n)}\right]'$ and $\mathbf{V} = \left[\mathrm{cov}(x_{(i)}, x_{(j)}\right]$ are expected value and covariance matrix of ordered statistics, respectively. The coefficients a_i are normalized, i.e. $\mathbf{a}'\mathbf{a} = 1$, and have the property $a_{n-i+1} = -a_i$, and, for odd n, $a_{\frac{n}{2}+1} = 0$. The exact elements of the matrix $\mathbf{V}$ are not known for large samples and algorithms for their evaluations are very memory and time consuming.

In the literature, different approximations to the exact a_i are considered, giving different normality tests. For example, in Shapiro and Francia (1972) test order statistics are assumed to be independent or in Weisberg and Bingham (1975) test $m_i = \Phi^{-1}\left(\frac{i - 0.125}{n + 0.25}\right)$ are additionally taken.

Many statistical books contain the tables of a_i given by Shapiro and Wilk (1965) for $n \le 50$ and recommend Royston's approximation (1982) for greater sample sizes (see for example Srivastava (2002), Thode (2002), Zielinski and Zielinski (1990)). However, they seem to ignore the paper by Royston (1992) in which the author points that "*Shapiro and Wilk's (1965) approximation for* *n>20 (Royston 1982 for n>50) is inadequate; even their exact values for* $n \le 20$ *are incorrect*."

Royston (1992) gives a new approximation for coefficients a_i and a normalizing transformation for the statistic *W* enabling its *p*–value computations for $4 \le n < 2000$. This Royston's method is implemented in the procedure "shapiro.test" in R program.

In the next section we show that coefficients a_{n-i+1} and critical values given by Shapiro and Wilk (1965) give Type I error a little beyond the nominal one. Then, we give tables for the a_{n-i+1} and critical values according to the Royston's approximation.

2. Type I error for Shapiro–Wilk W statistic

The Type I error for Shapiro–Wilk *W* test of normality have been evaluated by simulation study using R program. We generated 1 000 000 pseudorandom samples of the size *n* from normal distribution, and for each of them the value of the statistic *W* were calculated according to (1.1) with coefficients a_i by Shapiro and Wilk (1965). Next, the proportion of the values *W* which were less than the critical value given in Shapiro and Wilk (1965) were calculated.

The significance levels $\alpha = 0.01, 0.05, 0.1$ and sample sizes $n = 4(1)58$ were taken into account. The results rounded to the third decimal place are given in Table 1. It can be seen that Type I errors are a little beyond the nominal significance levels. In the case of $\alpha = 0.05$ it can be even 28% too low ($n = 5$). As there were 1 000 000 generated samples, the standard errors of the values in Table 1 are very small and equal approximately only 0.0001 for $\alpha = 0.01$, 0.0002 for $\alpha = 0.05$ and 0.0003 for $\alpha = 0.1$. Thus the results given in Table 1 seem to be reliable.

Not large departure, at first sight, from the nominal 0.05 can cause rather large change in the power of the test. For example, if sample of size $n = 50$ comes from t_3 distribution, the power of the test at $\alpha = 0.05$ is 0.54 (when a_i and critical value are taken after Shapiro and Wilk, 1965) while the power with coefficients and critical values after Royston (1992) is 0.64. Of course for distributions with light tails the relationship of the powers can be opposite, for example for $Beta(1,1)$ distribution the powers are 0.86 and 0.75, respectively.

Table 1. Type I errors for Shapiro–Wilk test for chosen sample sizes *n*

	Significance level α		
n	0.01	0.05	0.1
4	0.008	0.039	0.087
5	0.007	0.036	0.087
6	0.009	0.045	0.097
8	0.009	0.045	0.098
10	0.009	0.046	0.096
40	0.008	0.046	0.096
50	0.007	0.043	0.097

3. Tables according to P.R. Royston

Table 2 contains the coefficients a_{n-i+1} calculated according to the approximation given in Royston (1992).

Table 3 contains critical values for *W* with coefficients given in Table 2. The critical values were obtained by simulation study. For different sample size *n*, 50 000 pseudorandom samples from normal distribution were generated. For each of them the value of statistic *W* with coefficients given in Table 2 was calculated. The *p*–th quantiles of the values *W* are taken as the critical values at significance level $\alpha = p$.

Table 2. Coefficients a_{n-i+1} for the *W* statistic according to Royston (1992)

	n													
i	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	.6873	.6646	.6430	.6231	.6051	.5887	.5737	.5600	.5474	.5358	.5250	.5150	.5056	.4968
2	.1663	.2414	.2807	.3030	.3163	.3243	.3290	.3315	.3326	.3327	.3320	.3309	.3295	.3277
3			.0883	.1411	.1751	.1982	.2143	.2260	.2345	.2408	.2455	.2489	.2514	.2532
4					.0565	.0951	.1228	.1433	.1589	.1709	.1804	.1879	.1939	.1987
5							.0401	.0698	.0924	.1101	.1242	.1356	.1448	.1525
6									.0304	.0540	.0729	.0881	.1007	.1111
7											.0240	.0435	.0594	.0727
8													.0196	.0360
	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1	.4885	.4807	.4734	.4664	.4598	.4535	.4475	.4418	.4363	.4311	.3044	.4213	.4167	.4122
2	.3259	.3238	.3217	.3196	.3174	.3152	.3130	.3108	.3087	.3065	.2523	.3023	.3003	.2983
3	.2545	.2552	.2557	.2558	.2557	.2554	.2550	.2545	.2538	.2531	.2163	.2515	.2506	.2496
4	.2026	.2057	.2083	.2104	.2120	.2133	.2143	.2151	.2157	.2161	.1868	.2165	.2165	.2164
5	.1589	.1642	.1686	.1724	.1756	.1783	.1806	.1825	.1842	.1856	.1611	.1877	.1886	.1892
6	.1199	.1273	.1336	.1390	.1436	.1476	.1511	.1542	.1568	.1591	.1381	.1629	.1644	.1658
7	.0839	.0934	.1015	.1085	.1145	.1198	.1245	.1285	.1321	.1353	.1169	.1406	.1428	.1448
8	.0497	.0613	.0713	.0799	.0874	.0940	.0997	.1048	.1093	.1133	.0971	.1201	.1230	.1256
9	.0164	.0304	.0423	.0526	.0616	.0694	.0764	.0825	.0879	.0928	.0783	.1010	.1045	.1077
10			.0140	.0261	.0366	.0458	.0539	.0611	.0675	.0732	.0602	.0829	.0871	.0908
11					.0122	.0228	.0321	.0404	.0477	.0543	.0427	.0655	.0703	.0747
12							.0107	.0201	.0285	.0359	.0255	.0487	.0542	.0591
13									.0095	.0179	.0085	.0323	.0384	.0440
14												.0161	.0229	.0292
15													.0076	.0145

Table 2. continued

	n													
i	32	33	34	35	36	37	38	39	40	41	42	43	44	45
1	.4080	.4039	.3999	.3960	.3923	.3887	.3853	.3819	.3786	.3755	.3724	.3694	.3665	.3637
2	.2963	.2943	.2924	.2905	.2887	.2869	.2851	.2833	.2816	.2800	.2783	.2767	.2571	.2736
3	.2487	.2477	.2467	.2457	.2447	.2437	.2427	.2417	.2406	.2396	.2386	.2376	.2366	.2356
4	.2163	.2161	.2158	.2155	.2151	.2147	.2142	.2138	.2133	.2128	.2122	.2117	.2111	.2105
5	.1898	.1902	.1906	.1908	.1910	.1911	.1911	.1911	.1911	.1910	.1908	.1907	.1905	.1902
6	.1669	.1679	.1688	.1696	.1703	.1708	.1713	.1717	.1721	.1723	.1726	.1727	.1729	.1730
7	.1465	.1481	.1495	.1508	.1519	.1529	.1538	.1546	.1553	.1559	.1564	.1569	.1573	.1577
8	.1279	.1300	.1319	.1336	.1351	.1365	.1378	.1390	.1400	.1410	.1418	.1426	.1433	.1439
9	.1106	.1132	.1155	.1177	.1197	.1215	.1231	.1246	.1260	.1272	.1284	.1294	.1304	.1313
10	.0942	.0973	.1002	.1028	.1051	.1073	.1093	.1111	.1128	.1144	.1158	.1171	.1184	.1195
11	.0787	.0823	.0856	.0886	.0914	.0939	.0963	.0984	.1004	.1023	.1040	.1056	.1070	.1084
12	.0636	.0678	.0715	.0750	.0782	.0811	.0838	.0863	.0886	.0907	.0927	.0946	.0963	.0979
13	.0491	.0537	.0580	.0619	.0655	.0688	.0718	.0746	.0772	.0797	.0819	.0840	.0860	.0878
14	.0348	.0400	.0448	.0491	.0531	.0568	.0602	.0633	.0663	.0690	.0715	.0739	.0761	.0781
15	.0208	.0265	.0318	.0366	.0410	.0451	.0489	.0524	.0556	.0586	.0614	.0640	.0665	.0688
16	.0069	.0132	.0190	.0243	.0292	.0336	.0378	.0416	.0452	.0485	.0516	.0545	.0572	.0597
17			.0063	.0121	.0174	.0223	.0269	.0311	.0350	.0386	.0419	.0451	.0480	.0508
18					.0058	.0111	.0161	.0206	.0249	.0288	.0325	.0359	.0391	.0421
19							.0054	.0103	.0149	.0191	.0231	.0268	.0303	.0335
20									.0050	.0096	.0138	.0178	.0215	.0250
21											.0046	.0089	.0129	.0166
22													.0043	.0083

	n												
i	46	47	48	49	50	51	52	53	54	55	56	57	58
1	.3609	.3582	.3556	.3531	.3506	.3842	.3458	.3435	.3413	.3391	.3369	.3348	.3327
2	.2720	.2705	.2691	.2676	.2662	.2648	.2635	.2621	.2608	.2595	.2582	.2570	.2558
3	.2346	.2336	.2327	.2317	.2308	.2298	.2289	.2280	.2271	.2262	.2253	.2244	.2235
4	.2099	.2093	.2087	.2081	.2075	.2069	.2063	.2057	.2051	.2045	.2038	.2032	.2026
5	.1900	.1897	.1894	.1891	.1888	.1885	.1881	.1878	.1874	.1870	.1866	.1862	.1858
6	.1730	.1730	.1730	.1730	.1729	.1728	.1727	.1725	.1724	.1722	.1720	.1718	.1716
7	.1580	.1583	.1585	.1587	.1589	.1590	.1591	.1592	.1592	.1592	.1592	.1592	.1592
8	.1445	.1450	.1455	.1459	.1463	.1466	.1469	.1471	.1474	.1476	.1477	.1479	.1480
9	.1321	.1328	.1335	.1341	.1347	.1352	.1357	.1361	.1365	.1369	.1372	.1375	.1378
10	.1205	.1215	.1224	.1232	.1240	.1247	.1253	.1259	.1265	.1270	.1275	.1279	.1283
11	.1097	.1108	.1119	.1130	.1139	.1148	.1156	.1164	.1171	.1178	.1184	.1190	.1195
12	.0994	.1008	.1021	.1033	.1044	.1055	.1064	.1074	.1082	.1091	.1098	.1105	.1112
13	.0895	.0911	.0926	.0940	.0953	.0965	.0977	.0988	.0998	.1008	.1017	.1025	.1033
14	.0801	.0819	.0835	.0851	.0866	.0880	.0893	.0906	.0917	.0928	.0939	.0948	.0958
15	.0709	.0729	.0748	.0766	.0782	.0798	.0813	.0827	.0840	.0852	.0864	.0875	.0885
16	.0620	.0642	.0663	.0683	.0701	.0718	.0735	.0750	.0765	.0778	.0791	.0804	.0815
17	.0534	.0558	.0580	.0602	.0622	.0641	.0659	.0676	.0692	.0707	.0721	.0735	.0748
18	.0449	.0475	.0500	.0523	.0545	.0565	.0585	.0603	.0621	.0637	.0653	.0668	.0682
19	.0365	.0394	.0420	.0446	.0469	.0492	.0513	.0533	.0552	.0569	.0586	.0603	.0618
20	.0283	.0314	.0342	.0369	.0395	.0419	.0442	.0463	.0484	.0503	.0521	.0539	.0555
21	.0201	.0234	.0265	.0294	.0322	.0348	.0372	.0395	.0417	.0438	.0457	.0476	.0494
22	.0121	.0156	.0189	.0220	.0249	.0277	.0303	.0328	.0351	.0373	.0394	.0414	.0433
23	.0040	.0078	.0113	.0146	.0178	.0207	.0235	.0261	.0286	.0310	.0332	.0354	.0374
24			.0038	.0073	.0106	.0138	.0168	.0196	.0222	.0247	.0271	.0294	.0315
25					.0035	.0069	.0100	.0130	.0158	.0185	.0210	.0234	.0257
26							.0033	.0065	.0095	.0123	.0150	.0175	.0199
27									.0032	.0061	.0090	.0117	.0142
28											.0030	.0058	.0085
29													.0028

Table 3. Critical points for statistic *W*

	α			
n	0.01	0.02	0.05	0.1
4	.6931	.7176	.7612	.8007
5	.6969	.7284	.7759	.8120
6	.7187	.7510	.7930	.8285
7	.7368	.7665	.8085	.8401
8	.7570	.7846	.8214	.8515
9	.7685	.7968	.8335	.8614
10	.7844	.8102	.8449	.8704
11	.7968	.8212	.8546	.8784
12	.8094	.8326	.8624	.8853
13	.8188	.8424	.8708	.8924
14	.8279	.8495	.8763	.8971
15	.8369	.8558	.8816	.9011
16	.8409	.8609	.8867	.9058
17	.8504	.8685	.8921	.9097
18	.8552	.8741	.8960	.9131
19	.8619	.8796	.9008	.9166
20	.8657	.8837	.9043	.9198
21	.8731	.8884	.9079	.9230
22	.8769	.8915	.9112	.9253
23	.8813	.8949	.9135	.9275
24	.8833	.8979	.9171	.9302
25	.8873	.9012	.9190	.9325
26	.8915	.9046	.9214	.9340
27	.8938	.9066	.9236	.9363
28	.8957	.9088	.9527	.9380
29	.9002	.9132	.9287	.9401
30	.9037	.9158	.9308	.9417
31	.9061	.9179	.9317	.9426
32	.9094	.9204	.9341	.9444
33	.9103	.9212	.9352	.9456
34	.9121	.9225	.9365	.9468
35	.9146	.9246	.9385	.9484
36	.9162	.9259	.9399	.9495
37	.9186	.9282	.9410	.9501
38	.9199	.9298	.9423	.9515
39	.9223	.9316	.9437	.9529
40	.9232	.9324	.9444	.9533
41	.9247	.9337	.9457	.9544
42	.9272	.9362	.9469	.9553
43	.9280	.9372	.9481	.9563
44	.9290	.9377	.9487	.9570

	α			
n	0.01	0.02	0.05	0.1
45	.9293	.9381	.9491	.9574
46	.9331	.9409	.9513	.9587
47	.9324	.9406	.9513	.9589
48	.9343	.9424	.9520	.9597
49	.9353	.9431	.9531	.9604
50	.9371	.9442	.9539	.9613
51	.9376	.9450	.9548	.9619
52	.9390	.9462	.9559	.9626
53	.9402	.9473	.9561	.9630
54	.9404	.9476	.9570	.9636
55	.9422	.9488	.9577	.9645
56	.9420	.9492	.9582	.9646
57	.9430	.9498	.9587	.9650
58	.9446	.9511	.9594	.9657

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