# CROWN WIDTH OF A TREE AND ITS RELATIONSHIPS WITH AGE, HEIGHT AND DIAMETER AT BREAST HEIGHT BASED ON COMMON OAK (QUERCUS ROBUR L.) 

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#### Abstract

Summary

The paper presents results of studies on a relationship between crown width with basic measurements of the tree. The aim of the analysis was to determine the strength of the relationship between crown diameter of a tree and its diameter at breast height and height as well as age. Moreover, regression equations were developed for the estimation of crown width. The experimental material comprised measurement data of 33 oaks (aged from 41 to 148 years). In view of the statistically significant dependence between crown width of oaks and measured tree traits the analysis of regression was conducted, assuming the investigated traits (age, height and diameter at breast height) as explanatory variables. The forward stepwise regression was applied. Crown width of a tree may be determined both on the basis of information on the age, height and diameter at breast height of a tree. Diameter at breast height was the best among measurable traits of a tree for the estimation of crown width.


Key words and phrases: crown width, diameter at breast height, height, stepwise regression, oak (Quercus robur L.)

## 1. Introduction

The crown of a tree to a considerable extent determines its growth and increment. Extensive studies concerning the structure and shape of crowns in different tree species were conducted by Burger and Badoux (after Borowski 1974). The dependence of crown size on other traits of a tree was evaluated e.g. by Dubravac T., Krejci V. (1993), Dubravac T. (1998, 1999, 2003, 2004), and Hemery et al. (2005). There are no studies in Polish literature on the size of the crown in oaks and its effect on tree growth. Analyses concerning the crown were conducted mainly in pine stands (Lemke 1966).
The aim of this study was:

1) To determine the strength of the relationship between crown diameter of a tree and the basic measurable traits, such as diameter at breast height and the height of a tree as well as its age,
2) To develop regression equations for the estimation of crown width.

## 2. Experimental material and methods

Material for analyses comprises results of measurements of 33 oaks (aged from 41 to 148 years) coming from 11 stands (the Łopuchówko Forest Division, the Regional Directorate of State Forests in Poznań). In each of the 11 stands a 1-hectare experimental plot was established, in each of which 3 mean sample trees were selected according to the principles of variant I of the Urich method (diameter classes). Measurements taken on mean sample trees were used to determine the course of growth and increment of basic mensuration traits. Additionally selected characteristics of crown dimensions were determined on standing trees. Crown projection area was established on the basis of characteristic crown points projected using a tree crown profile projector. Crowns were projected in eight geographical directions (N, N-W, W, W-S, S, S-E, E, E-N). Radiuses were measured from the stem centre to the projected point. Crown projection area was calculated as a sum of the areas of triangles defining it. Crown width and its radius expressed in metres were obtained from the crown projection area assumed as the area of a circle.

The method of least squares was used to estimate parameters of equations and the strength of the relationship was established on the basis of empirical data. The function which best explained the dependence of crown width on breast height diameter was selected. Moreover, the distribution of measurement data was compared with the normal distribution and their statistical characteristics were established. The strength of the relationship between crown width and diameter at breast height, height and age was evaluated by the
correlation coefficient for a linear dependence. In turn, the strength of the dependence between crown width and diameter at breast height was evaluated on the basis of a correlation ratio for curvilinear functions. The correlation ratio was determined by a formula (Bruchwald 1994):

$$
\begin{equation*}
R=\sqrt{1-\frac{\sum\left(y_{i}-\hat{y}_{i}\right)^{2}}{\sum\left(y_{i}-\bar{y}\right)^{2}}} \tag{2.1}
\end{equation*}
$$

where:

$$
\begin{array}{ll}
\sum\left(y_{i}-\hat{y}_{i}\right)^{2} & \text { - remainder sum of squares of deviations, } \\
\sum\left(y_{i}-\bar{y}\right)^{2} & \text { - total sum of squares of deviations. }
\end{array}
$$

## 3. Results

Basic statistical characteristics of analysed trees were established (tab. 1). Age of trees was found to be within a broad range from 41 to 148 years, with mean age of 92 years. Mean height was 24.8 m . The lowest tree was 18.3 m tall, while the tallest was 34.3 m . Height was characterised by the lowest variation, as it only slightly exceeded $20 \%$. Mean diameter at breast height was 31.48 cm , ranging from 14.55 cm to 56.85 cm . Crown width in oaks was on average 6.17 m , at a coefficient of variation of $39.02 \%$.

Table 1. A characteristic of selected measurable traits of trees

| Trait | N | $\bar{x}$ | $\min$ | $\max$ | $s_{d x}$ | $\mathrm{~V}(\%)$ |
| :---: | ---: | :---: | :--- | :---: | ---: | ---: |
| $w$ (age - years) | 33 | 92 | 41 | 148 | 33.96 | 37.11 |
| $h$ (height -m$)$ | 33 | 24.8 | 18.3 | 34.3 | 5.05 | 20.29 |
| $d_{z k}($ diameter at breast height <br> outside bark -cm$)$ | 33 | 31.48 | 14.55 | 56.85 | 11.17 | 35.49 |
| $d k$ (crown width -m$)$ | 33 | 6.17 | 3.04 | 14.01 | 2.41 | 39.02 |

Coefficients of a linear correlation between four traits of trees: age, diameter at breast height, height and crown width, were calculated. Results are given in Table 2. All correlation coefficients are significant at 0.05.

The biggest correlation coefficient was found for the dependence between crown width and diameter at breast height, amounting to 0.869 . A slightly weaker dependence was recorded between diameter at breast height and height (0.843). An even weaker relationship was found between diameter at breast height and age (0.802). Height is correlated with age at 0.771 , while with crown
width it was only slightly lower (0.749). The weakest dependence was found for the relationship of crown width with age (0.675).

Table 2. A correlation diagram

| Trait | $w$ | $h$ | $d$ | $d k$ |
| :---: | :---: | :---: | :---: | :---: |
| $w$ |  | 0.771 | 0.802 | 0.675 |
| $h$ | 0.771 |  | 0.843 | 0.749 |
| $d$ | 0.802 | 0.843 |  | 0.869 |
| $d k$ | 0.675 | 0.749 | 0.869 |  |

Parameters of equations were determined for the above mentioned linear dependencies. Three equations of linear regression were obtained for the determination of crown width in oaks, which assumed the following forms:
$\begin{array}{ll}\text { a) depending on age } & d k=0.0478 \cdot w+1.7912 \\ \text { b) depending on height } & d k=0.3568 \cdot h-2.6945 \\ \text { c) depending on diameter at breast height } & d k=0.1873 \cdot d+0.2739 .\end{array}$
Fluctuations in crown width depending on age, height and diameter at breast height in the analysed trees with the course of the fitted regression line are presented in Fig. 1.

In view of the statistically significant relationship between crown width of oaks and measurable traits of trees the analysis of regression was conducted assuming all the analysed traits (age, height and diameter at breast height) as independent variables. Forward stepwise regression was applied. Initially the following form of the equation was obtained:

$$
\begin{equation*}
d k=-0.152-0.006 \cdot w+0.040 \cdot h+0.187 \cdot d \tag{3.4}
\end{equation*}
$$

The goodness of fit for this regression equation was $74.8 \%$, at a coefficient of multiple correlation of 0.871 . However, not all parameters of this equation were statistically significant. After stepwise regression was performed a previously known linear equation (3.3) was finally obtained, where crown width depends only on breast height diameter of a tree.


Fig. 1. Fluctuations in crown width depending on age (a), height (b) and diameter at breast height (c).
Next parameters of curvilinear equations (quadratic polynomials, power and exponential functions) were estimated using the method of least squares. Equations for the estimation of crown width took the following form:
quadratic polynomial

$$
\begin{equation*}
d k=0.0002 \cdot d^{2}+0.1732 \cdot d+0.4935 \tag{3.5}
\end{equation*}
$$

power function

$$
\begin{equation*}
d k=0.2530 \cdot d^{0.9224} \tag{3.6}
\end{equation*}
$$

exponential function

$$
\begin{equation*}
d k=2.3205 \cdot e^{0.289 \cdot d} \tag{3.7}
\end{equation*}
$$

The strength of the relationship determined by the correlation ratio for these functions generally does not diverge from the linear correlation coefficient (Tab. 3).

Table 3. Strength of fit for empirical data in estimated regression equations

| Equation | Correlation <br> coefficient | Correlation <br> ratio |
| :---: | :---: | :---: |
| $(3.3)$ | 0.8690 |  |
| $(3.5)$ |  | 0.8961 |
| $(3.6)$ |  | 0.8665 |
| $(3.7)$ |  | 0.8541 |

It is also confirmed by the graphic presentation of fitting the estimated equations to empirical data (Fig. 2). The course of individual functions in terms of measured data was similar. It differed only slightly at extreme values.


Fig. 2. A dependence of crown width on diameter at breast height on the basis of different functions (——polynomial, ..........power, -----exponential and..-- .. -linear)

## 4. Discussion

A relationship between crown width of a tree and the diameter of this tree measured at a height of 1.3 m has been confirmed in numerous studies (e.g. Avsar 2004, Avsar, Ayyildiz 2005, Akalp 1983, Bragg 2001, Gering, May 1995, Sun 1977, Hasenauer 1997). Another measure of the crown has also been used in research, i.e. crown radius (Franscis 1988, Foli et al. 2003). In the presented investigations the strongest relationship was recorded between crown width and diameter at breast height, while it was slightly weaker for crown width with height. The strength of the relationship of breast height diameter with height classifies it among the above mentioned relationships. A similar dependence established on the basis of the correlation coefficient was found in case of cedar
trees (Avsar, Ayyildiz 2005). Avsar (2004) in pine (Pinus brutia Ten.) stated a strong dependence of crown width with diameter at breast height, at a correlation coefficient of 0.86 . Similar results were recorded in the case of cedar trees (Cedrus libani A. Rich.), where variation in diameter at breast height was explained by the variation of crown width in $72.2 \%$ (Avsar, Ayyildiz 2005). This dependence is described by a rectilinear model, which most frequently is the best (e.g. Cañadas 2000, Paulo et al. 2002, Benítez et al. 2003). It was also described by a quadratic polynomial, a power function or a monomolecular function (Avsar 2004, Bechtold 2003, Bragg 2001, Sánchez-González et al. 2007, Tomé et al. 2001).

## 5. Conclusions

1) Crown width of a tree may be determined both on the basis of age, height and diameter at breast height of a tree.
2) The best measurement trait of a tree for the estimation of crown width is the diameter at breast height. This is indicated by the strength of a dependence between both traits and the easy measurement of tree diameter at a height of 1.3 m .

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