# THE APPROXIMATION OF TREE VOLUME BASED ON MEASUREMENTS OF BIOMETRIC FEATURES OF STANDING SCOTS PINE TREES 

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#### Abstract

Summary The paper presents results of studies on a dependency between tree volume on basic stem measurements - dbh, height, double bark thickness, period increment in breast height diameter, period increment in height, age of trees and slenderness. Regression equations were developed for the estimation of tree volume. Material for analyses comprises results of measurements of 200 pine coming from 8 stands. All stands from which the experimental test trees derived grew on the fresh mixed coniferous forest sites situated in the Zielonka Experimental Forest District. Mean sample trees from the sample plot following the methodology developed by Draudt. In view of the statistically significant dependence between volume of pine and measured tree traits the analysis of regression was conducted, assuming the investigated traits (diameter at breast height, height, double bark thickness, 5 -year increment in breast height diameter, 5 -year increment in height, slenderness, age of trees) as explanatory variables. The backward stepwise regression was applied. Tree volume may be determined both on the basis of information on the height and diameter at breast height of a tree, double bark thickness and slenderness.


Keywords and phrases: diameter at breast height, height, stepwise regression, Scots pine (Pinus sylvestris L.)

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## 1. Introduction

The production of wood is the basic function of a forest. The improvement the methods of calculating tree and stand volume is fundamental for forest mensuration. Growth and increment of a tree as well as factors affecting its size and variation belong to the basic fields of research in forestry (Assmann 1968, Borowski 1974). Insight into changes occurring in the growth of trees contributes to an appropriate course of silvicultural procedures conducted in the forest management unit (Jaworski 2004). Research in this field has been conducted primarily in relation to Scots pine. This situation is perfectly understandable as it is the main forest-forming species in Poland. The fluctuations in growth and volume increment or selected volume elements have been analyzed. Growth dynamics in pine have been investigated by e.g. Szymański (1963), Żółciak (1963), Ważyński (1967), Borowski, Grochowski (1969) and Lemke (1971, 1972a, 1972b, 1984, 1988).

Estimation of a stem volume is an important aspect of a forest mensuration. Stem volume is function of a tree's height, diameter, bark thickness, shape. It is one of the most difficult parameters to measure. The most accurately method of estimating tree volume is sectional method. Stem volume derived from summation of sections. The tree is sectioned into a identical length sections, the dimensions of these are measured, section volumes are derived, and stem volume is obtained by summation (Grochowski 1973, Bruchwald 1999). A direct evaluation of the sectional volume of standing trees is quite difficult and simply impossible. The improvement the methods of calculating tree volume is fundamental of forest mensuration. The volume and the measures used to derive it (breast height diameter, height, increment of dbh, height increment, slenderness) are most important tree and stand parameters. Slenderness is the ratio of height in m to breast height diameter in cm . Tree slenderness is one of the shape characteristics of longitudinal cross-section of the stem (Bruchwald 1999, Grochowski 1973, Jaworski 2004). Slenderness coefficient is regarded as measure of the stability of (Zajaczkowski 1991). Regression function for tree volume have been developed as a function of easily measurable tree variables on standing tree. The volume models should be based on the variables that are usually measured in forest inventories or which be estimated easily.

The aim of this study was:

- To determine the strength of the relationship between tree dimensions and tree volume,
- To develop regression equations for the estimation of tree volume.


## 2. Experimental material and methods

The experimental material included selected analysis results of 200 pinetree trunks derived from 8 stands. All stands from which the experimental test trees derived grew on the fresh mixed coniferous forest sites situated in the Zielonka Experimental Forest District. Mean sample trees from the sample plot following the methodology developed by Draudt (Grochowski 1973). In the study measurement traits were used:

- $w$ - age of sample trees (years),
- $d_{1.3}-$ breast height diameter $(\mathrm{cm})$,
- $h$ - tree height ( m ),
- $Z d_{5}-5$-year increment in breast height diameter (cm),
- $Z h_{5}-5$-year increment in height (m),
- $k$ - double bark thickness (measured in cm at the height of 1.3 m ),
- $s$ - slenderness (defined as the ratio of height in m to breast height diameter in cm ),
- $\quad V$ - tree volume $\left(\mathrm{m}^{3}\right)$.

The tree volume based on section measure volume in $\mathrm{m}^{3}$. The stem is divided into sections of identical length - volume of each complete section is established as the volume of a cylinder with a diameter measured in mid-length of a given section, while the volume of the last incomplete section is calculated using a formula for the volume of a cone; such calculated volumes of individual sections are summed.

Most of the analyzed traits were determined for standing trees, whereas $Z h_{5}$ and $V$ were established on felled trees.

## 3. Results

The analysed trees represented 8 age subclasses (the age of 24 to 92 years). Diameter at breast height of sample trees ranged from 6.96 to 43.13 cm , with an average of 21.01 cm and a coefficient of variation $37.43 \%$ (Table 1). Less variability was notable for the height $(22.95 \%)$, with the arithmetic mean of 20.91 m . The double bark thickness was characterized by the high variation - 56.73\% (Table 1). Double bark thickness ranged from 0.33 to 6.70 cm , with an average area of 2.20 cm . The 5 -year increment in breast height diameter reached from 0.17 to 4.25 cm , with an average width equal to 1.32 cm . The coefficient of variation of this trait was $50.83 \%$. The 5 -year increment in height ranged from 0.27 to 3.75 m , with an average equal 1.62 m (Table 1). The slenderness reached
a level from 0.78 to 1.936 with an average slenderness of 1.18 . The tree volume was characterized by the highest variation $-83.65 \%$ (Table 1). Tree volume ranged from 0.02 to $1.54 \mathrm{~m}^{3}$, with an average area of $0.39 \mathrm{~m}^{3}$.

Table 1. The characteristic of selected measurable traits of trees

| Trait | N | $\bar{x}$ | $\operatorname{Min}$ | $\max$ | $s_{d x}$ | $\mathrm{v}(\%)$ |
| :---: | :---: | ---: | ---: | ---: | ---: | :---: |
| $w$ (age of stand - years) | 200 | 58.13 | 24.00 | 92.00 | 22.58 | 38.85 |
| $d_{1.3}$ (diameter at breast height - cm) | 200 | 21.01 | 6.96 | 43.13 | 7.86 | 37.43 |
| $h$ (height- m) | 200 | 20.91 | 11.38 | 29.26 | 4.80 | 22.95 |
| $k$ (double bark thickness -cm$)$ | 200 | 2.20 | 0.33 | 6.70 | 1.25 | 56.73 |
| $Z d_{5}(5-$-year increment in breast <br> height diameter -cm$)$ | 200 | 1.32 | 0.17 | 4.25 | 0.67 | 50.83 |
| $Z h_{5}(5$-year increment in height - m) | 200 | 1.62 | 0.27 | 3.75 | 0.77 | 47.38 |
| $s=h / d_{1.3}($ slenderness) | 200 | 1.18 | 0.78 | 1.93 | 0.24 | 19.87 |
| $V\left(\right.$ tree volume $\left.-\mathrm{m}^{3}\right)$ | 200 | 0.39 | 0.02 | 1.54 | 0.33 | 83.65 |

The dependences of pine volume on selected biometric characteristics was examined. Coefficients of linear correlation were calculated. The tree volume showed statistically significant correlation with all the traits (Table 2). With the growth of tree age, diameter at breast height, height, double bark thickness, 5 -year increment in breast height diameter volume increases. The strongest linear association of the volume was found with the thickness of trees (0.9645), the weaker with the height (0.8585), age (0.7597), double bark thickness ( 0.7501 ) and slenderness ( -0.7409 ). With the growth of tree slenderness volume decreases. The lowest strength was obtained for the 5-year increment in height $(-0.5248)$, and 5-year increment in breast height diameter (0.3574).

Table 2. The table of correlation

| Trait | $w$ | $d_{1.3}$ | $h$ | $k$ | $Z d_{5}$ | $Z h_{5}$ | $s$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $d_{1.3}$ | $0.7894^{*}$ |  |  |  |  |  |  |
| $h$ | $0.9023^{*}$ | $0.9015^{*}$ |  |  |  |  |  |
| $k$ | $0.6593^{*}$ | $0.7988^{*}$ | $0.6667^{*}$ |  |  |  |  |
| $Z d_{5}$ | $-0.0684^{*}$ | $0.4132^{*}$ | $0.1623^{*}$ | $0.2289^{*}$ |  |  |  |
| $Z h_{5}$ | $-0.8363^{*}$ | $-0.5149^{*}$ | $-0.6841^{*}$ | $-0.4311^{*}$ | $0.3613^{*}$ |  |  |
| $s$ | $-0.4522^{*}$ | $-0.8344^{*}$ | $-0.6131^{*}$ | $-0.5956^{*}$ | $-0.6829^{*}$ | 0.1186 |  |
| $V$ | $\mathbf{0 . 7 5 9} *$ | $\mathbf{0 . 9 6 4 5} *$ | $\mathbf{0 . 8 5 8 5} *$ | $\mathbf{0 . 7 5 0 1 *}$ | $\mathbf{0 . 3 5 7 4 *}$ | $\mathbf{- 0 . 5 2 4 8} *$ | $\mathbf{- 0 . 7 4 0 9 *}$ |

*The correlation coefficient significant at the level $\alpha=0.05$

The tree volume was also describes by multiple empirical equations. All the previously mentioned biometric features were included as the explanatory variables. Stepwise regression was performed, which step by step creates the best regression model (Krzyśko 2000). Backward stepwise regression was used, which sequentially removes from the model those variables which in every step have the least significant impact on the response variable - the volume. In the backward stepwise regression predictors remove from the model on the base of multivariate Wilks' Lambda test. This test can be interpreted as the multivariate generalization of a univariate R -squared, that is, it indicates the proportion of generalized variance in the dependent variables that is accounted for by the predictors (Stanisz 2007). At each step highly correlated predictors become redundant and are removed from the model on the base of indicators such as tolerances $\left(1-R^{2}\right)$. The "best" regression model contains predictors having the greatest influence on dependent variable (Jolliffe 1982).

The four equations to determine the volume of pine were given as a result of estimation the parameters (Table 3).

Table 3. Parameters of equations

| Equation | Equation parameters |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | intercept | $w$ | $d_{1.3}$ | $h$ | $k$ | $Z d_{5}$ | $Z h_{5}$ | $s$ |  |
| $(3.1)$ | $-1.412^{*}$ | 0.000 | $0.084^{*}$ | $-0.032^{*}$ | $-0.060^{*}$ | -0.012 | 0.021 | $0.692^{*}$ |  |
| $(3.2)$ | $-1.405^{*}$ |  | $0.084^{*}$ | $-0.032^{*}$ | $-0.059^{*}$ | -0.012 | 0.019 | $0.690^{*}$ |  |
| $(3.3)$ | $-1.425^{*}$ |  | $0.03^{*}$ | $-0.031^{*}$ | $-0.058^{*}$ |  | 0.014 | $0.704^{*}$ |  |
| $(3.4)$ | $-1.320^{*}$ |  | $0.082^{*}$ | $-0.033^{*}$ | $-0.058^{*}$ |  |  | $0.671^{*}$ |  |

*The parameter significant at the level $\alpha=0.05$
Volume of pine can be determined by regression equation in dependence to the diameter at breast height, height, double bark thickness and slenderness. All of the analyzed traits were determined for standing trees (Table 4).

Table 4. Multiple and partial correlation coefficients for the tree volume dependence on the selected characteristics of trees

| Equation | $\mathrm{R}_{\text {multiple }}$ | $\mathrm{R}_{\text {partial }}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $w$ | $d_{1.3}$ | $h$ | $k$ | $Z d_{5}$ | $Z h_{5}$ | $s$ |
| $(3.1)$ | 0.9715 | 0.021 | $0.904^{*}$ | $-0.583^{*}$ | $-0.566^{*}$ | -0.077 | 0.126 | $0.717^{*}$ |
| $(3.2)$ | 0.9715 |  | $0.904^{*}$ | $-0.640^{*}$ | $-0.576^{*}$ | -0.082 | 0.135 | $0.720^{*}$ |
| $(3.3)$ | 0.9713 |  | $0.97^{*}$ | $-0.637^{*}$ | $-0.573^{*}$ |  | 0.113 | $0.737^{*}$ |
| $(3.4)$ | 0.9710 |  | $0.907^{*}$ | $-0.670^{*}$ | $-0.571^{*}$ |  |  | $0.755^{*}$ |

[^0]Among the regression equations obtained from stepwise regression procedure, the best is the one with the small number of predictors but as good as others with respect to R multiple coefficient.

The sslenderness is defined as the ratio of height to breast height diameter so the obtained best model has the form of (3.4) equation in Table 3.

$$
V=-1.320+0.082 \cdot d_{1,3}-0.033 \cdot h-0.058 \cdot k+0.671 \cdot \frac{h}{d_{1.3}}
$$

## 4. Discussion

Scots pine is the most important predominating species forming forests in Poland. Stem volume is function of a tree's height, basal area.

For estimating tree volume are direct and indirect methods. The direct methods tend to divide the stem into sections and measure the volume of these sections. The indirect methods include volume tables and volume equations. A tree volume tables is a statement of the expected volume of a tree volume of a tree of nominated dimensions in a particular stand or population. Standard tables are with two dimensions: dbh and height. In Poland foresters use generally Czuraj's "Tables of volume for standing trees".

Foresters have developed relationships to estimate stem volume from measurements made on diameter at breast height and tree height. Volume equations are developed now. Regression function for tree volume have been developed as a function of easily measurable tree variables on standing tree.

## 5. Conclusions

- With the growth of diameter at breast height, height, age, double bark thickness, 5-year increment in breast height diameter pine volume increases.
- With the growth of tree slenderness and 5-year increment in height pine volume decreases.
- The strongest association of the volume was found with the diameter at breast height and tree height.
- Volume of pine can be determined by regression equation (3.4) of dependence on the diameter at breast height, height and double bark thickness.


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[^0]:    *The correlation coefficient significant at the level $\alpha=0.05$

