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RELATIONS BETWEEN RANDOMIZED BLOCK DESIGNS AND WEIGHING DESIGNS IN EXAMPLES

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Summary

In this paper the relations between a randomized block designs and spring balance weighing designs are presented. The main idea is to present these relations from the point of view of examples of agricultural experiments.

Keywords and phrases: spring balance weighing design, optimal design, randomized block design

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1. Introduction

When we want to compare all treatments in similar conditions, the experimental units are arranged in compact sets called blocks. Because the treatments were assigned to units within blocks at random, the design was called "randomized blocks" (Cochran and Cox, 1957). Following Caliński and Kageyama (2000), we present used names: "the randomized-blocks design" (Scheffé, 1959), "the randomized block design" (Finney, 1960; Hinkelmann and Kempthorne, 1994; Raghavarao and Padgett, 2005), "the randomized complete block design" (Federer, 1955). As well, the last one is presented in Montgomery (2013).

Let us consider model of a block design given in the form

$$\mathbf{y}^* = \mathbf{\Delta}'\boldsymbol{\tau} + \mathbf{D}'\boldsymbol{\beta} + \mathbf{e}^*, \tag{1.1}$$

where $\mathbf{y}^*$ is the $n^* \times 1$ vector of observations, $\mathbf{\Delta}'$ is $n^* \times v^*$ design matrix for treatments, $\mathbf{D}'$ is $n^* \times b^*$ design matrix for blocks, $\boldsymbol{\tau}$ is the $v^* \times 1$ vector of treatment parameters, $\boldsymbol{\beta}$ is the $b^* \times 1$ vector of block parameters, $\mathbf{e}^*$ is the $n^* \times 1$ random vector of errors. We assume that $\mathrm{E}(\mathbf{e}^*) = \mathbf{0}_{n^*}$, $\mathrm{Var}(\mathbf{e}^*) = (\sigma^*)^2 \mathbf{I}_{n^*}$, where $\mathbf{I}_s$ denotes $s \times s$ identity matrix.

We will use the following notation: $\mathbf{N}^* = \mathbf{\Delta D}'$ is called the incidence matrix, $\mathbf{N}^* \mathbf{1}_{b^*} = \mathbf{r}^* = (r_1^*, r_2^*, ..., r_{v^*}^*)'$, $\mathbf{R}^* = \mathrm{diag}(r_1^*, r_2^*, ..., r_{v^*}^*)$, $(\mathbf{N}^*)' \mathbf{1}_{v^*} = \mathbf{k}^* = (k_1^*, k_2^*, ..., k_{b^*}^*)'$, $\mathbf{K}^* = \mathrm{diag}(k_1^*, k_2^*, ..., k_{b^*}^*)$, $\mathbf{\Gamma} = \mathbf{I}_{n^*} - \mathbf{D}'(\mathbf{K}^*)^{-1}\mathbf{D}$, $\mathbf{1}_s$ denotes $s \times 1$ vector of ones. The matrix $\mathbf{U}$ from reduced set of normal equations is given as $\mathbf{U} = \mathbf{R}^* - \mathbf{N}^*(\mathbf{K}^*)^{-1}(\mathbf{N}^*)' + \frac{1}{n^*}\mathbf{r}^*(\mathbf{r}^*)'$, $\mathbf{U}^{-1} = \mathbf{\Omega}$. We consider the model of block design without general mean. In order to estimate the vector of treatment parameters $\boldsymbol{\tau}$ we solve the reduced set of normal equations under assumption that $(\mathbf{k}^*)'\boldsymbol{\beta} = 0$. Thus the estimator obtained by the least squares method $\hat{\boldsymbol{\tau}}$ is equal to $\mathbf{By}^*$, where $\mathbf{B} = \mathbf{\Omega\Delta}\left(\frac{1}{n^*}\mathbf{1}_{n^*}\mathbf{1}_{n^*}' + \mathbf{\Gamma}\right)$. With this notation $\hat{\boldsymbol{\tau}}$ is unbiased estimator of $\boldsymbol{\tau}$, i.e. $\mathrm{E}(\mathbf{By}^*) = \boldsymbol{\tau}$, $\hat{\boldsymbol{\tau}} = \boldsymbol{\tau} + \mathbf{e}$ and $\mathbf{e} = \mathbf{Be}^*$. An easy computation shows that $\mathrm{Var}(\mathbf{e}) = \mathrm{Var}(\mathbf{Be}^*) = \mathbf{B}\,\mathrm{Var}(\mathbf{e}^*)\mathbf{B}' = (\sigma^*)^2\mathbf{\Omega}$.

Following Ceranka and Katulska (1990), we assume that we can express $\boldsymbol{\tau}$ as $\boldsymbol{\tau} = \mathbf{Xw}$, where $\mathbf{w}$ is the $p \times 1$ vector of unknown parameters and $\mathbf{X}$ is an $v^* \times p$ matrix of elements $x_{ij} = 1$ or 0. It means $\mathbf{X}$ is the matrix of factorial design with p factors. The element x_{ij} equal to 1 or 0 indicates the existence or absence of respectively factor. Then

$$\hat{\boldsymbol{\tau}} = \mathbf{Xw} + \mathbf{e}, \tag{1.2}$$

where $\mathrm{E}(\mathbf{e}) = \mathbf{0}_{v^*}$ and $\mathrm{Var}(\mathbf{e}) = (\sigma^*)^2 \mathbf{\Omega}$. On the other hand, $\mathbf{X}$ is the design matrix of the spring balance weighing design and (1.2) we can treat as the model of spring balance weighing design when $v^* = n$. The problem is how to choose the combinations of p factors, i.e. to determine matrix $\mathbf{X}$ which is optimal in some sense. In the special case, when an experiment is implemented in a randomized block design $\mathbf{\Omega} = (r^*)^{-1} \mathbf{I}_{v^*}$, the study of relations between these models was presented by Federer et al. (1976) and Ceranka and Katulska (1987).

In the theory of weighing designs, the problem is to estimate the unknown elements of the vector $\mathbf{w}$ in proper sense by minimizing some functions of the matrix $\mathbf{M} = \mathbf{X}'\mathbf{X}$ called the information matrix for the design $\mathbf{X}$. Here, we consider the most popular in agriculture experiments optimality criterion: A-optimality and seldom used E-optimality.

Definition 1.1 Any design $\mathbf{X}_{\mathrm{A}}$ is A-optimal if $\mathrm{tr}(\mathbf{M}_{\mathrm{A}}^{-1})$ is minimal in the class of all possible design matrices $\mathbf{X}_d$. Moreover, if $\mathrm{tr}(\mathbf{M}_{\mathrm{A}}^{-1})$ attains the lower bound then the design is called the regular A-optimal.

Definition 1.2. Any design $\mathbf{X}_{\mathrm{E}}$ is regular E-optimal if $\lambda_{\max}(\mathbf{M}_{\mathrm{E}}^{-1})$ is minimal in the class of all possible design matrices $\mathbf{X}_d$. Moreover, if $\lambda_{\max}(\mathbf{M}_{\mathrm{E}}^{-1})$ attains the lower bound then the design is called the regular E-optimal.

The purpose of the paper is to present on the base of some examples the relations between the experimental plans of the randomized block designs and spring balance weighing designs. In next sections two agricultural experiments are discussed.

2. Example of the A-optimal design

Let us consider the experiment in that the influence of various organic fertilizers, applied under the fore crop in relation to NPK (nitrogen, phosphorus, potassium), on stem-root diseases of spring barley as consecutive crop was examined. The field experiment was carried out in years 1998-2000 in Experimental Station Mydlniki. The results of experiment are presented by Boligłowa and Łabza (2001). The experiment was found in randomized complete block design with four replications. The influences of NPK and different kinds of organic mass on the health condition of strew of spring barley were compared. The infection of spring barley stem base by pathogen depending on organic fertilization applied under the fore crop is presented in the Table 1.

Table 1. Infection of spring barley stem base by pathogen *Gaeumannomyces graminis* depending on organic fertilization applied under the fore crop

Treatment	*Gaeumannomyces graminis*
NPK	15.89
NPK + white mustard	19.67
Fresh dung	24.25
Fresh dung + white mustard	20.46
Triticale straw	23.06
Triticale straw + white mustard	26.96
Spring barley straw	30.62
Spring barley straw + white mustard	23.21
White flowering pea straw	23.60
White flowering pea straw + white mustard	25.73

According to the model (1.1), for randomized block design, we have $\mathbf{y}^*$ is the 40×1 vector of observations, $n^* = 40$, $b^* = r^* = 4$, $v^* = k^* = 10$, $\mathbf{\Delta}' = \mathbf{I}_{10} \otimes \mathbf{1}_4$, $\mathbf{D}' = \mathbf{1}_{10} \otimes \mathbf{I}_4$, $\mathbf{B} = 0.25(\mathbf{I}_{10} \otimes \mathbf{1}'_4)$. Thus the result of experiment described according to the model (1.2) we write as

$$\begin{bmatrix} 15.89 \\ 19.67 \\ 24.25 \\ 20.46 \\ 23.06 \\ 26.96 \\ 30.62 \\ 23.21 \\ 23.60 \\ 25.73 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix}.$$

The first column of the matrix $\mathbf{X}$ corresponds to the influence of NPK w_1, the second to the white mustard w_2, the third one to fresh dung w_3, w_4 to the triticale straw, w_5 to the spring barley straw and the last one w_6 to the white flowering pea straw. Next

$$\hat{\mathbf{w}} = \mathbf{M}^{-1}\mathbf{X}'\mathbf{y} = \begin{bmatrix} 17.919 & -0.278 & 22.494 & 25.149 & 27.054 & 24.804 \end{bmatrix}'$$

and

$$\mathrm{Var}(\hat{\mathbf{w}}) = \sigma^2\mathbf{M}^{-1} = \sigma^2 \begin{bmatrix} 0.6 & -0.2 & 0.1 & 0.1 & 0.1 & 0.1 \\ -0.2 & 0.4 & -0.2 & -0.2 & -0.2 & -0.2 \\ 0.1 & -0.2 & 0.6 & 0.1 & 0.1 & 0.1 \\ 0.1 & -0.2 & 0.1 & 0.6 & 0.1 & 0.1 \\ 0.1 & -0.2 & 0.1 & 0.1 & 0.6 & 0.1 \\ 0.1 & -0.2 & 0.1 & 0.1 & 0.1 & 0.6 \end{bmatrix}.$$

Thus $\mathrm{Var}(\hat{w}_2) = 0.4\sigma^2$ and $\mathrm{Var}(\hat{w}_1) = \mathrm{Var}(\hat{w}_3) = \mathrm{Var}(\hat{w}_4) = \mathrm{Var}(\hat{w}_5) = \mathrm{Var}(\hat{w}_6) = 0.6\sigma^2$. As we are interested in comparing above experimental plan with the plan determined by using the regular A-optimal spring balance weighing design, we take the matrix $\mathbf{M}$ and we obtain $\mathrm{tr}(\mathbf{M}^{-1}) = 3.4$.

The question is: could we reduce the variance of estimators by using some other experimental plan?

For improving the statistical properties of the experiment we suggest to take components considered in the experiment in some other combinations. For this purpose let us consider the design matrix of the regular A-optimal spring balance weighing design $\mathbf{X}_A$ given by Graczyk (2012) with the property that $\mathbf{X}_A = \begin{bmatrix} \mathbf{N}_1' \\ \mathbf{N}_2' \end{bmatrix}$. Here, $\mathbf{N}_1$ is the incidence matrix of the group divisible design with the parameters $v = 6$, $b_1 = 4$, $r_1 = 2$, $k_1 = 3$, $\lambda_{11} = 0$, $\lambda_{21} = 1$ (SR18) and $\mathbf{N}_2$ is the design matrix of the group divisible design with the parameters $v = 6$, $b_2 = 6$, $r_2 = 3$, $k_2 = 3$, $\lambda_{12} = 2$, $\lambda_{22} = 1$ (S42), where

$$\mathbf{N}_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \ \mathbf{N}_2 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

For details of this construction we refer the reader to Graczyk (2012), Lemma 3.2(a) for $t = 1$. We follow the notation given by Clatworhty (1973) writing SR18 and S42. We did not put into practice the experiment, so we are now in a position to present (1.2) in the form $\hat{\boldsymbol{\tau}} = \mathbf{X}_A \mathbf{w} + \mathbf{e}$ as

$$\boldsymbol{\tau} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix},$$

$$\mathrm{Var}(\hat{\mathbf{w}}) = \sigma^2 \mathbf{M}_A^{-1} = \frac{\sigma^2}{45} \begin{bmatrix} 13 & -2 & -2 & -2 & -2 & -2 \\ -2 & 13 & -2 & -2 & -2 & -2 \\ -2 & -2 & 13 & -2 & -2 & -2 \\ -2 & -2 & -2 & 13 & -2 & -2 \\ -2 & -2 & -2 & -2 & 13 & -2 \\ -2 & -2 & -2 & -2 & -2 & 13 \end{bmatrix}.$$

In this way we obtain $\mathrm{Var}(\hat{w}_j) = 0.2(8)\sigma^2$, $j = 1,2,...,6$. It means we estimate the influences of NPK, white mustard, fresh dung, triticale straw, spring barley straw and white flowering pea straw with the same variance $0.2(8)\sigma^2$, which is smaller than the one given in experiment. $\mathbf{X}_A$ is the design matrix of the regular A-optimal spring balance weighing design, so the trace of the inverse of information matrix is equal $\mathrm{tr}(\mathbf{M}_A)^{-1} = 1.7(3)$ and takes the smallest value. So, using the regular A-optimal weighing design we are able to reduce the variance of the estimators almost twice.

3. Example of the E-optimal design

Now, let us consider the experiment described in the paper of Drzewiecki and Pietryga (2002). In 2000-2001 an experiment was carried out to study the influence of application of growth regulator + foliar fertilizers mix used in winter wheat on the quality of grain. The experiment was found in the Institute of Plant Protection in Sośnicowice. The content of protein in winter wheat grain in 2001 is presented in the Table 2.

Table 2. Effect of tank-mix application of growth regulator and foliar fertilizers on content of protein of winter wheat grain in 2001

Treatment	Content of protein (%)
Growth regulator	11.6
Growth regulator + Basfoliar 36 Extra	12.4
Growth regulator + Basfoliar 34	12.4
Growth regulator + Basfoliar 12-4-6	12.0
Growth regulator + Insol 3	11.7
Growth regulator + Mikrosol Z	11.7
Growth regulator + Wuxal Top N	12.5
Growth regulator + Wuxal 36	11.9

The problem we wish to study is to present the relations between randomized complete block designs and regular E-optimal spring balance weighing designs. For this reason, in the Table 2, there is presented the part of results given in paper Drzewiecki and Pietryga (2002).

For randomized complete block design, according to the model (1.1), we have $\mathbf{y}^*$ is the 32×1 vector of observations, $n^* = 32$, $b^* = r^* = 4$, $v^* = k^* = 8$, $\mathbf{\Delta}' = \mathbf{I}_8 \otimes \mathbf{1}_4$, $\mathbf{D}' = \mathbf{1}_8 \otimes \mathbf{I}_4$, $\mathbf{B} = 0.25(\mathbf{I}_8 \otimes \mathbf{1}_4')$. Thus the result of experiment described according to the model (1.2) we write as

$$\begin{bmatrix} 11.6 \\ 12.4 \\ 12.4 \\ 12.0 \\ 11.7 \\ 11.7 \\ 12.5 \\ 11.9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \end{bmatrix}.$$

The first column of the matrix $\mathbf{X}$ corresponds to the influence of growth regulator w_1, the second to the influence of Basfoliar 36 Extra w_2, the third one to Basfoliar 34 w_3, w_4 to the Basfoliar 12-4-6, w_5 to the Insol 3, w_6 to the Mikrosol Z, w_7 to the Wuxal Top N and finally w_8 to the Wuxal 36. Then we obtain the estimator of unknown influences of components $\hat{\mathbf{w}} = \mathbf{M}^{-1}\mathbf{X}'\mathbf{y} = [\,11.6 \quad 0.8 \quad 0.8 \quad 0.4 \quad 0.1 \quad 0.1 \quad 0.9 \quad 0.3\,]'$ with the variance

$$\mathrm{Var}(\hat{\mathbf{w}}) = \sigma^2\mathbf{M}^{-1} = \sigma^2 \begin{bmatrix} 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 \\ -1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \end{bmatrix}.$$

Thus the biggest variance of the estimators is $\mathrm{Var}(\hat{w}_j) = 2\sigma^2$, $j = 2,3,...,8$. It is therefore of interest the comparison of the variance of estimators in above experiment and in the experiment arranged by applying the regular E-optimal spring balance weighing design. Finally, according to the definition of E-optimality, we count $\lambda_{\max}(\mathbf{M}^{-1}) = 0.5(9 + \sqrt{77}) = 8.887$.

Now, we determine the design for that the maximal eigenvalue of $\mathbf{M}^{-1}$ is minimal. It is equivalent to the determining the regular E-optimal spring balance weighing design. In this way we will reduce the variance of estimators. The matrix of experimental plan given by Drzewiecki and Pietryga (2002) we can write in the form $\mathbf{X} = \begin{bmatrix} 1 & \mathbf{0}_7' \\ \mathbf{1}_7 & \mathbf{X}_1 \end{bmatrix}$, $\mathbf{X}_1 = \mathbf{I}_7$, here $\mathbf{0}_s$ denotes $s \times 1$ vector of zeros. This form indicates that in each measurement we take growth regulator and because of this the elements of first column are equal to 1. We are not able to give the construction of the matrix $\mathbf{X}_\mathrm{E}$ (8×8) which is regular E-optimal and has the analogous structure. So, the proposal of new structure is related to the matrix $\mathbf{X}_1$ (7×7), however the conclusions regard the matrix $\mathbf{X}$ (8×8).

In order to get the design with better statistical properties it is convenient to consider the design matrix $\mathbf{X}_\mathbf{E}$ of the regular E-optimal spring balance weighing design given in Ceranka and Graczyk (2014), Theorem 3.3(iv) for $t = 1$. $\mathbf{X}_\mathrm{E} = \mathbf{N}'$, where $\mathbf{N}$ is the incidence matrix of balanced incomplete block design with the parameters $v = b = 7$, $r = k = 3$, $\lambda = 1$ in the form

$$\mathbf{N} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{X}_\omega = \begin{bmatrix} 1 & \mathbf{0}_7' \\ \mathbf{1}_7 & \mathbf{X}_\mathrm{E} \end{bmatrix}.$$

We are able to write

$$\boldsymbol{\tau} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \end{bmatrix}$$

and, moreover,

$$\operatorname{Var}(\hat{\mathbf{w}}) = \sigma^2 \mathbf{M}_{\omega}^{-1} = \frac{\sigma^2}{18}\begin{bmatrix} 18 & -6 & -6 & -6 & -6 & -6 & -6 & -6 \\ -6 & 10 & 1 & 1 & 1 & 1 & 1 & 1 \\ -6 & 1 & 10 & 1 & 1 & 1 & 1 & 1 \\ -6 & 1 & 1 & 10 & 1 & 1 & 1 & 1 \\ -6 & 1 & 1 & 1 & 10 & 1 & 1 & 1 \\ -6 & 1 & 1 & 1 & 1 & 10 & 1 & 1 \\ -6 & 1 & 1 & 1 & 1 & 1 & 10 & 1 \\ -6 & 1 & 1 & 1 & 1 & 1 & 1 & 10 \end{bmatrix}.$$

Afterwards, $\operatorname{Var}(\hat{w}_1) = \sigma^2$ and $\operatorname{Var}(\hat{w}_j) = 0.(5)\sigma^2$ for $j = 2,3,\ldots,8$. Furthermore, $\lambda_{\max}\left(\mathbf{M}_{\omega}^{-1}\right) = \frac{1}{18}\left(17 + \sqrt{253}\right) = 1.8$. It is worth underlying that the minimizing of the maximal eigenvalue of the matrix $\mathbf{M}^{-1}$ is equivalent to determining the design in that the maximal variance of estimators is minimal. It is obvious that using regular E-optimal spring balance weighing design as experimental plan we are able to reduce the variance of estimator almost four times. It is worth noting that taking measurements in proposed combinations we have been working under assumption that there is no interaction between foliar fertilizers. Moreover, in practical experiment the doses of these fertilizers have to be reconsider.

4. Discussion

The above considerations may be summarized by saying that it seems to be profitable to use the plans of optimal weighing designs as the scheme for selection the components to the mixtures. By using the randomized complete block designs we are able to compare the mean values of mixtures. By using weighing designs we are able to compare the influence of each from considered components. Moreover, using regular A-optimal and E-optimal spring balance weighing designs we obtain estimators with observable smaller variance. Obviously, from statistical point of view such plans have better statistical properties.

The principal significance of application of weighing designs as experimental plans is obtaining more information about separately influence of considered components. We are able to answer the question which of the considered components has the biggest influence on the experimental results. So, we are able to improve the analysis of the experiment.

The paper should be treated as suggestion for the next studies. Obviously, in both experiments we take different mixtures of components. The mixtures of components are strongly depended on the experimental conditions, so we need to reconsider the doses.

References

Boligłowa E., Łabza T. (2001). Sequent effect of organic fertilizers on stem-root diseases of spring barley. *Progress in Plant Protection / Postępy w Ochronie Roślin* 41 (2), 688−691.

Caliński T., Kageyama S. (2000). *Block Designs: A randomized Approach, V 1: Analysis*. Springer, New York.

Ceranka B., Graczyk M. (2014). Regular E-optimal spring balance weighing designs with correlated errors. *Communication in Statistics-Theory and Methods* 43, 947−953.

Ceranka B., Katulska K. (1987). Zastosowanie teorii sprężynowych układów wagowych do analizy doświadczeń z mieszankami. *Listy Biometryczne* 24, 17−26.

Ceranka B., Katulska K. (1990). Optimum biased spring balance weighing designs under equal correlations of errors. *Biometrical Journal* 8, 931−942.

Clatworthy W.H. (1973). *Tables of Two-Associate-Class Partially Balanced Design*. NBS Applied Mathematics Series 63.

Cochran W.G., Cox G.M. (1957). *Experimental Designs.* 2nd ed. Wiley, New York.

Drzewiecki S., Pietryga J. (2002). Influence of growth regulator with foliar fertilizers on quality of winter wheat grain. *Progress in Plant Protection / Postępy w Ochronie Roślin* 42 (2), 522−525.

Federer W.T. (1955). *Experimental Design: Theory and Application*. Macmillan, New York.

Federer W.T., Hedayat A., Love C.C., Raghavarao D. (1976). Application of statistical design theory to crop estimation with special reference to legumes and mixtures of cultivars. *Agron. J.* 68, 914−919.

Graczyk M. (2012). Regular A-optimal spring balance weighing designs. *Revstat* 10, 3, 323−333.

Hinkelmann K., Kempthorne O. (1994). *Design and Analysis of Experiments, Vol. I: Introduction to Experimental Design*. Wiley, New York.

Montgomery D.C. (2013). *Design and Analysis of Experiments.* John Wiley & Sons, Inc., Singapore.

Raghavarao D., Padgett L.V. (2005). *Block Designs, Analysis, Combinatorics and Applications.* Series of Applied Mathematics 17, Word Scientific Publishing Co. Pte. Ltd., Singapore.

Scheffé H. (1959). *The Analysis of Variance*. Wiley, New York.