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D-OPTIMAL DESIGNS WITH NEGATIVE CORRELATED ERRORS BASED ON TERNARY DESIGNS: CONSTRUCTION

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Summary

Here, we study new construction methods of the regular D-optimal chemical balance weighing design with equally negative correlated errors. The construction method is based on the set of the incidence matrices of the ternary balanced block designs.

Key words and phrases: chemical balance weighing design, D-optimality, ternary balanced block design

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1. Introduction

The aim of this paper is to present some problems related to the chemical balance weighing designs. Let $\mathbf{y}$ be a $n\times 1$ random vector of the observations. If $\mathbf{y} = \mathbf{Xw} + \mathbf{e}$, where $\mathbf{w} = \left(w_1, w_2, ..., w_p\right)'$ is vector representing unknown measurements of p objects, $\mathbf{X} \in \mathbf{\Phi}_{n\times p, m}\{-1, 0, 1\}$, $\mathbf{\Phi}_{n\times p, m}\{-1, 0, 1\}$ denotes the class of $n\times p$ matrices having entries $-1, 1$ or 0, m $(\leq n)$ is the maximal number of elements equal to 1 and –1 in each column of the matrix $\mathbf{X}$, then the respectively model is called chemical balance weighing design. Here, $\mathbf{e}$ is a $n \times 1$ random vector of errors. In the literature, different assumptions regard to the form of its variance are considered. Throughout this paper the following assumptions will be needed: $\mathrm{E}(\mathbf{e}) = \mathbf{0}_n$ and $\mathrm{Var}(\mathbf{e}) = \sigma^2 \mathbf{G}$, where

$$\mathbf{G} = g\left((1-\rho)\mathbf{I}_n + \rho \mathbf{1}_n \mathbf{1}_n^{'}\right), \quad g > 0, \quad \frac{-1}{n-1} < \rho < 0. \tag{1.1}$$

The form of matrix $\mathbf{G}$ indicates that the errors of measurements are equally correlated and they have the same variances. Note that for $\frac{-1}{n-1} < \rho < 1$ and $g > 0$ the matrix $\mathbf{G}$ is positive definite. If the matrix $\mathbf{X}$ is of full column rank then the estimator of the vector $\mathbf{w}$ is given in the form $\hat{\mathbf{w}} = \mathbf{M}^{-1}\mathbf{X}^{'}\mathbf{G}^{-1}\mathbf{y}$ and its covariance equals $\sigma^2 \mathbf{M}^{-1}$, where $\mathbf{M} = \mathbf{X}'\mathbf{G}^{-1}\mathbf{X}$ is called the information matrix for the design $\mathbf{X}$.

In the literature, several problems connected with weighing designs are discussed. Among these ones, Jacroux et al. (1983) and Sathe and Shenoy (1990) gave a brief discussion of different optimality criteria. Besides, Graczyk (2012) presents some issues concerned on A-optimality. Many questions regard to the D-optimality are considered in Smaga (2014) and Ceranka and Graczyk (2014a). In the papers: Li and Yang (2005), Neubauer and Pace (2010), Ceranka and Graczyk (2014b), the attention is focus on the constructions of the design matrices of the optimal designs. Besides, in Koukouvinos and Seberry (1997), Banerjee and Mukerjee (2007) some applications of such designs are indicated.

In the theory of experimental designs, different optimality criteria are considered. The optimality criteria minimize some functions of information matrix for the design. Here, we have been working under assumption that we minimize the general variance, i.e. we determine D-optimal design. The design $\mathbf{X}_D$ is called D-optimal in the given class $\mathbf{\Phi}_{n \times p,m}\{-1, 0, 1\}$ if $\det\left(\mathbf{X}_D^{'}\mathbf{G}^{-1}\mathbf{X}_D\right)^{-1} = \min\left(\det\left(\mathbf{M}^{-1}\right) : \mathbf{X} \in \mathbf{\Phi}_{n \times p,m}\{-1, 0, 1\}\right)$. If $\det(\mathbf{X}_D)$ attains the lowest bound then the design is called regular D-optimal. In other cases, it is called D-optimal. It is evident; each regular D-optimal design is D-optimal whereas the inverse sentence may be not true. Raghavarao (1971), Shah and Sinha (1989), Masaro and Wong (2008), Katulska and Smaga (2013), Neubauer and Pace (2010), Smaga (2014) presented many the results connected with D-optimality. Particularly, some aspects of the regular D-optimal chemical balance weighing design under assumption that the errors are correlated and they have equal variances, i.e. in $\sigma^2\mathbf{G}$ the matrix $\mathbf{G}$ is as in (1.1), are presented by Ceranka and Graczyk (2014a). We remind the definition of the D-optimal design and the theorem determining the parameters of the regular D-optimal design given in this paper.

Definition 1.1. Any chemical balance weighing design $\mathbf{X} \in \mathbf{\Phi}_{n\times p,m}\{-1, 0, 1\}$ with the covariance matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is given by (1.1), is regular D-optimal if $\det(\mathbf{M}^{-1}) = \left(g(1-\rho)\left(m - \frac{\rho(m-2u)^2}{1+\rho(n-1)} \right)^{-1} \right)^p$, where $u = \min\{u_1, u_2, ..., u_p\}$, u_j represent the number of elements equal to -1 in j th column of $\mathbf{X}$, $j = 1,2,..., p$.

Theorem 1.1. Any chemical balance weighing design $\mathbf{X} \in \mathbf{\Phi}_{n\times p,m}\{-1, 0, 1\}$ with the covariance matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is of (1.1), is regular D-optimal if and only if

$$\mathbf{X}'\mathbf{X} = m\mathbf{I}_p - \frac{\rho(m-2u)^2}{1+\rho(n-1)}\left(\mathbf{I}_p + \mathbf{1}_p\mathbf{1}_p'\right) \tag{1.2}$$

and

$$\mathbf{X}'\mathbf{1}_n = \mathbf{z}_p, \tag{1.3}$$

where $\mathbf{z}_p$ is $p \times 1$ vector for which the j th element is equal to $-(m-2u)$ or $m-2u$, $j = 1,2,..., p$.

Next, we will touch a few aspects of the construction methods of optimal designs.

2. The construction

Let $\mathbf{\Pi}_{n\times p}\{-1, 1\}$ denotes the class of the design matrices of the chemical balance weighing designs under basic assumption that the elements of the design matrix are equal to -1 or 1. In this case, some constructions of regular D-optimal designs were given in Masaro and Wong (2008) and Katulska and

Smaga (2013). Ceranka and Graczyk (2014b) considered the problem of D-optimality in weighing designs under the more general assumption that $\mathbf{X} \in \mathbf{\Phi}_{n\times p,m}\{-1, 0, 1\}$. In this paper, the list of the parameters of the regular D-optimal chemical balance weighing design is given. Notwithstanding, not for any pair of numbers: objects p and measurements n, we are able to give the construction method of D-optimal chemical balance weighing design which is based on the method presented in Ceranka and Graczyk (2014b). For that reason, in present paper, we study new construction method of D-optimal chemical balance weighing design $\mathbf{X} \in \mathbf{\Phi}_{n\times p,m}\{-1, 0, 1\}$. This method utilizes the incidence matrices of the ternary balanced block designs (see Billington, 1984). Accordingly, let us recall basic properties of such designs. Any ternary balanced block design there is an arrangement of v treatments in b blocks, each of size k in such a way that each treatment appears 0, 1 or 2 times in r blocks. Each of the distinct pairs of treatments appears λ times. Any ternary balanced block design is regular, that is, each treatment occurs alone in ρ_1 blocks and is repeated two times in ρ_2 blocks, where ρ_1 and ρ_2 are constant for the design. It is easy to check that

$$vr = bk,\ r = \rho_1 + 2\rho_2,\ \lambda(v-1) = \rho_1(k-1) + 2\rho_2(k-2).$$

For the incidence matrix $\mathbf{N}$, we obtain $\mathbf{N}\mathbf{N}' = (\rho_1 + 4\rho_2 - \lambda)\mathbf{I}_v + \lambda\mathbf{1}_v\mathbf{1}_v'$.

Let $\mathbf{N}_h$ be the incidence matrix of ternary balanced block design with the parameters v, b_h, r_h, k_h, λ_h, ρ_{1h}, ρ_{2h}, $h = 1,2,...,t$. We construct the design matrix $\mathbf{X} \in \mathbf{\Phi}_{n\times p,m}\{-1, 0, 1\}$ of the chemical balance weighing design as

$$\mathbf{X} = \begin{bmatrix} \mathbf{N}_1' - \mathbf{1}_{b_1}\mathbf{1}_v' \\ \mathbf{N}_2' - \mathbf{1}_{b_2}\mathbf{1}_v' \\ \cdots \\ \mathbf{N}_t' - \mathbf{1}_{b_t}\mathbf{1}_v' \end{bmatrix}. \tag{2.1}$$

Each column of $\mathbf{X}$ contains $\sum_{h=1}^{t}\rho_{2h}$ elements equal to 1, $\sum_{h=1}^{t}(b_h - \rho_{1h} - \rho_{2h})$ elements equal to –1 and $\sum_{h=1}^{t}\rho_{1h}$ elements equal to 0. It is obvious that each object is weighted $m = \sum_{h=1}^{t}\left(b_h - \rho_{1h}\right)$ times in the $n = \sum_{h=1}^{t} b_h$ weighing operations.

In order to determine the estimator of the vector $\mathbf{w}$ in the form $\hat{\mathbf{w}} = \mathbf{M}^{-1}\mathbf{X}^{'}\mathbf{G}^{-1}\mathbf{y}$, we need to prove the following Lemma.

Lemma 2.1. Any chemical balance weighing design with the design matrix $\mathbf{X} \in \mathbf{\Phi}_{n\times p,m}\{-1, 0, 1\}$ given by (2.1) is nonsingular if and only if $v \neq k_h$ for at least one $h = 1,2,...,t$.

Proof. If the matrix $\mathbf{G}$ is positive definite, then $\mathbf{M}$ is nonsingular if and only if $\mathbf{X}^{'}\mathbf{X}$ is nonsingular. Thus, for $\mathbf{G}$ in (1.1) and $\mathbf{X} \in \mathbf{\Phi}_{n\times p,m}\{-1, 0, 1\}$ given in (2.1), we have $\mathbf{X}^{'}\mathbf{X} = \left[\sum_{h=1}^{t}\left(r_h - \lambda_h + 2\rho_{2h}\right)\right]\mathbf{I}_v + \left[\sum_{h=1}^{t}\left(b_h + \lambda_h - 2r_h\right)\right]\mathbf{1}_v\mathbf{1}_v^{'}$.

Afterwards, we obtain $\det\left(\mathbf{X}^{'}\mathbf{X}\right) = \left[\sum_{h=1}^{t}\left(r_h - \lambda_h + 2\rho_{2h}\right)\right]^{v-1}\left[\sum_{h=1}^{t}\frac{r_h}{k_h}\left(v - k_h\right)^2\right]$.

For the parameters of any ternary balance block design, $r - \lambda + 2\rho_2 > 0$. From here, $\det\left(\mathbf{X}^{'}\mathbf{X}\right) = 0$ if and only if $v = k_h$ for each $h = 1,2,...,t$. In other words, the lemma is proved. ■

Theorem 2.1. Any nonsingular chemical balance weighing design $\mathbf{X} \in \mathbf{\Phi}_{n\times p,m}\{-1, 0, 1\}$ given by (2.1) with the covariance matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is given by (1.1), is regular D-optimal chemical balance weighing design if and only if the conditions mentioned below are simultaneously fulfilled

(i) $$\rho = \frac{\sum_{h=1}^{t}\left(b_h + \lambda_h - 2r_h\right)}{\left(\sum_{h=1}^{t}\left(r_h - b_h\right)\right)^2 - \left(\sum_{h=1}^{t} b_h - 1\right)\left(\sum_{h=1}^{t}\left(b_h + \lambda_h - 2r_h\right)\right)},$$

(ii) $\sum_{h=1}^{t}\left(b_h + \lambda_h - 2r_h\right) < 0$ and

(iii) $\sum_{h=1}^{t}(r_h - b_h) \neq 0$.

Proof. From Theorem 1.1, it follows that if $\frac{-1}{n-1} < \rho < 0$ then the chemical balance weighing design is regular D-optimal if and only if the conditions (1.2) and (1.3) hold. Formula $\mathbf{X}'\mathbf{X} = m\mathbf{I}_p - \frac{\rho(m-2u)^2}{1+\rho(n-1)}\left(\mathbf{I}_p - \mathbf{1}_p\mathbf{1}_p'\right)$ implies that $\mathbf{c}_j'\mathbf{X}'\mathbf{X}\mathbf{c}_j = \frac{\rho(m-2u)^2}{1+\rho(n-1)}$, where $\mathbf{c}_j$ is the j^{th} column of matrix $\mathbf{I}_p$. From the proof of Lemma 2.1 we obtain that $\mathbf{c}_j'\mathbf{X}'\mathbf{X}\mathbf{c}_j = \sum_{h=1}^{t}(b_h + \lambda_h - 2r_h)$. Thus $\sum_{h=1}^{t}(b_h + \lambda_h - 2r_h) = \rho\left(\sum_{h=1}^{t}(r_h - b_h)\right)^2\left(1+\rho\left(\sum_{h=1}^{t}b_h - 1\right)\right)^{-1}$ and from here ρ is of the form (i). If ρ is of the form (i), then $\sum_{h=1}^{t}(b_h + \lambda_h - 2r_h) < 0$. On the other hand, if $\sum_{h=1}^{t}(b_h + \lambda_h - 2r_h) < 0$ then the denominator of (i) is greater than zero and in that case $\frac{-1}{n-1} < \rho < 0$. Thus the condition (ii) has to be fulfilled. ■

There is a big number of combinations between the parameters of the ternary balanced block designs. Firstly, let us consider the case $t = 1$, i.e. $\mathbf{X} = \mathbf{N}' - \mathbf{1}_b\mathbf{1}_v'$, $\mathbf{X} \in \mathbf{\Phi}_{b\times v,\, b-\rho_1}\{-1, 0, 1\}$.

Theorem 2.2. If for a given ρ, the parameters of the ternary balanced block designs are equal to

(i) $\rho = -2(24s+13)^{-1}$, $v = 4s+1$, $b = 3(4s+1)$, $r = 12s$, $k = 4s$, $\lambda = 12s-5$, $\rho_1 = 4s$, $\rho_2 = 4s$, $s = 1,2,\ldots$,

(ii) $\rho = -(8s^2+1)^{-1}$, $v = 4s^2-1$, $b = 2(4s^2-1)$, $r = 4(2s^2-1)$,

$k = 2(2s^2 - 1)$, $\lambda = 8s^2 - 7$, $\rho_1 = 2(2s^2 - 1)$, $\rho_2 = 2s^2 - 1$, $s = 1,2,...$,

(iii) $\rho = -(8s+9)^{-1}$, $v = 4s+3$, $b = 2(4s+3)$, $r = 4(2s+1)$, $k = 2(2s+1)$, $\lambda = 8s+1$, $\rho_1 = 2(2s+1)$, $\rho_2 = 2s+1$, $s = 1,2,...$,

(iv) $\rho = -(16s+7)^{-1}$, $v = 8s+7$, $b = 2(8s+7)$, $r = 4(4s+3)$, $k = 2(4s+3)$, $\lambda = 16s+9$, $\rho_1 = 2(4s+3)$, $\rho_2 = 4s+3$, $s = 1,2,...$,

(v) $\rho = -(s^2+7s+5)^{-1}$, $v = 5$, $b = 5(s+1)$, $r = 4(s+1)$, $k = 4$, $\lambda = 3s+2$, $\rho_1 = 4s$, $\rho_2 = 2$, $s = 1,2,...$,

(vi) $\rho = -(26)^{-1}$, $v = 12$, $b = 18$, $r = 15$, $k = 10$, $\lambda = 11$, $\rho_1 = 1$, $\rho_2 = 7$,

then $\mathbf{X} = \mathbf{N}' - \mathbf{1}_b \mathbf{1}_v' \in \mathbf{\Phi}_{b\times v,\, b-\rho_1}\{-1, 0, 1\}$ is the regular D-optimal chemical balance weighing design with the covariance matrix of errors $\sigma^2\mathbf{G}$.

Proof. The parameters of the ternary balanced block design (i)-(vi) satisfy the conditions of Theorem 2.1. ■

Now, we consider the case $t = 2$. Next, we give a few corollaries presenting the series of the parameters of the ternary balanced block designs. Base on these parameters we construct appropriate incidence matrices of the ternary balanced block designs and next the design matrices of the regular D-optimal chemical balance weighing design with the covariance matrix of errors $\sigma^2\mathbf{G}$ in (1.1). It is obvious, that the parameters of the ternary balanced block design satisfy the conditions given in Theorem 2.1.

Corollary 2.1. If the parameters of the ternary balanced block design are equal to $v = 2s+1$, $b_1 = 4s+u+1$, $r_1 = 4s+u+1$, $k_1 = 2s+1$, $\lambda_1 = 4s+u-1$, $\rho_{11} = u+1$, $\rho_{21} = 2s$ and $v = 2s+1$, $b_2 = 2q(2s+1)$, $r_2 = 4q(s+1)$, $k_2 = 2(s+1)$, $\lambda_2 = 2q(2s+3)$, $\rho_{12} = 4qs$, $\rho_{22} = 2q$, $s,q = 1,2,...$, then $\mathbf{X} \in \mathbf{\Phi}_{(4s(q+1)+2q+u+1)\times(2s+1),\,2(2s+q)}\{-1, 0, 1\}$ given by (2.1) is the regular D-optimal chemical balance weighing design with the covariance matrix of errors $\sigma^2\mathbf{G}$ for $\rho = -\dfrac{1}{2q^2 + 4qs + 2q + 4s + u}$.

Corollary 2.2. If the parameters of the ternary balanced block design are equal to $v = 2s+1$, $b_1 = 2(2s+1)$, $r_1 = 2(2s+1)$, $k_1 = 2s+1$, $\lambda_1 = 4s$, $\rho_{11} = 2$, $\rho_{21} = 2s$ and $v = 2s+1$, $b_2 = 2q(2s+1)$, $r_2 = 4q(s+1)$, $k_2 = 2(s+1)$, $\lambda_2 = 2q(2s+3)$, $\rho_{12} = 4qs$, $\rho_{22} = 2q$, $s = 2,3,\ldots$, $q = 1,2,\ldots$, then $\mathbf{X} \in \mathbf{\Phi}_{2(q+1)(2s+1)\times(2s+1),2(2s+q)}\{-1, 0, 1\}$ given by (2.1) is the regular D-optimal chemical balance weighing design with the covariance matrix of errors $\sigma^2\mathbf{G}$ for $\rho = -\dfrac{1}{4s + 4qs + 2q^2 + 2q + 1}$.

Corollary 2.3. If the parameters of the ternary balanced block design are equal to $v = 2s+1$, $b_1 = 2(2s+1)$, $r_1 = 2(2s+1)$, $k_1 = 2s+1$, $\lambda_1 = 4s$, $\rho_{11} = 2$, $\rho_{21} = 2s$ and $v = 2s+1$, $b_2 = u(2s+1)$, $r_2 = u(2s-q-1)$, $k_2 = 2s-q-1$, $\lambda_2 = u(2s-2q-3)$, $\rho_{12} = u(2s-7q-1)$, $\rho_{22} = 3qu$, $s = 2,3,\ldots$, $q = 1,2,\ldots$, then $\mathbf{X} \in \mathbf{\Phi}_{(2s+1)(u+2)\times(2s+1),4s+u(7q+2)}\{-1, 0, 1\}$ given by (2.1) is the regular D-optimal chemical balance weighing design with the covariance matrix of errors $\sigma^2\mathbf{G}$ for $\rho = -\dfrac{2}{u^2q^2 + 4u^2q + 4u^2 + 8s + 4us + 2u + 1}$.

Corollary 2.4. If the parameters of the ternary balanced block design are equal to $v = 4s+1$, $b_1 = 3(4s+1)$, $r_1 = 12s$, $k_1 = 4s$, $\lambda_1 = 12s-5$, $\rho_{11} = 4s$, $\rho_{21} = 4s$ and $v = 4s+1$, $b_2 = 2q(4s+1)$, $r_2 = 4q(2s+1)$, $k_2 = 2(2s+1)$, $\lambda_2 = 2q(4s+3)$, $\rho_{12} = 8qs$, $\rho_{22} = 2q$, $s = 2,3,\ldots$, $q = 1,2,\ldots$, then $\mathbf{X} \in \mathbf{\Phi}_{(2q+1)(4s+1)\times(4s+1),8s+2q+3}\{-1, 0, 1\}$ given by (2.1) is the regular D-optimal chemical balance weighing design with the covariance matrix of errors $\sigma^2\mathbf{G}$ for $\rho = -\dfrac{2}{24s + 16qs + 4q^2 - 8q + 13}$.

Corollary 2.5. If the parameters of the ternary balanced block design are equal to $v = 4s+1$, $b_1 = 3(4s+1)$, $r_1 = 12s$, $k_1 = 4s$, $\lambda_1 = 12s-5$, $\rho_{11} = 4s$, $\rho_{21} = 4s$ and $v = 4s+1$, $b_2 = 2(8s+4q-5)$, $r_2 = 2(8s+4q-5)$,

$k_2 = 4s+1$, $\lambda_2 = 4(4s+2q-3)$, $\rho_{12} = 2(4s+4q-5)$, $\rho_{22} = 4s$, $s = 2,3,...$, $q = 1,2,...$, then $\mathbf{X} \in \mathbf{\Phi}_{(28s+8q-7)\times(4s+1),16s+3}\{-1, 0, 1\}$ given by (2.1) is the regular D-optimal chemical balance weighing design with the covariance matrix of errors $\sigma^2\mathbf{G}$ for $\rho = -\frac{4}{112s+32q-23}$.

Corollary 2.6. If the parameters of the ternary balanced block design are equal to $v = 4s+1$, $b_1 = 3(4s+1)$, $r_1 = 12s$, $k_1 = 4s$, $\lambda_1 = 12s-5$, $\rho_{11} = 4s$, $\rho_{21} = 4s$ and $v = 4s+1$, $b_2 = u(4s+1)$, $r_2 = u(4s-q-1)$, $k_2 = 4s-q-1$, $\lambda_2 = u(4s-2q-3)$, $\rho_{12} = u(4s-7q-1)$, $\rho_{22} = 3qu$, $s = 4,5,...$, $q, u = 1,2,...$, then $\mathbf{X} \in \mathbf{\Phi}_{(4s+1)(u+3)\times(4s+1),8s+3+u(7q+2)}\{-1, 0, 1\}$ given by (2.1) is the regular D-optimal chemical balance weighing design with the covariance matrix of errors $\sigma^2\mathbf{G}$ for

$$\rho = -\frac{2}{4u^2 + u^2q^2 + 4u^2q + 8su + 24s + 6uq + 14u + 13}.$$

Corollary 2.7. If the parameters of the ternary balanced block design are equal to $v = 4s+3$, $b_1 = 2(4s+3)$, $r_1 = 4(2s+1)$, $k_1 = 2(2s+1)$, $\lambda_1 = 8s+1$, $\rho_{11} = 2(2s+1)$, $\rho_{21} = 2s+1$ and $v = 4s+3$, $b_2 = 2q(4s+3)$, $r_2 = 8q(s+1)$, $k_2 = 4(s+1)$, $\lambda_2 = 2q(4s+5)$, $\rho_{12} = 4q(2s+1)$, $\rho_{22} = 2q$, $s = 4,5,...$, $q = 1,2,...$, then $\mathbf{X} \in \mathbf{\Phi}_{2(q+1)(4s+3)\times(4s+3),2(2s+q+1)}\{-1, 0, 1\}$ given by (2.1) is the regular D-optimal chemical balance weighing design with the covariance matrix of errors $\sigma^2\mathbf{G}$ for $\rho = -\frac{1}{4q^2-2q+8s+8qs+9}$.

Corollary 2.8. If the parameters of the ternary balanced block design are equal to $v = 4s+3$, $b_1 = 2(4s+3)$, $r_1 = 4(2s+1)$, $k_1 = 2(2s+1)$, $\lambda_1 = 8s+1$, $\rho_{11} = 2(2s+1)$, $\rho_{21} = 2s+1$ and $v = 4s+3$, $b_2 = 2(4qs+3)$, $r_2 = 2(4qs+3)$, $k_2 = 4s+3$, $\lambda_2 = 4(2qs+1)$, $\rho_{12} = 8(q-1)s+2$, $\rho_{22} = 2(2s+1)$, $s, q = 1,2,...$, then $\mathbf{X} \in \mathbf{\Phi}_{4(2s(q+1)+3)\times(4s+3),4(3s+2)}\{-1, 0, 1\}$ given by (2.1) is the regular D-optimal chemical balance weighing design with the covariance matrix of errors $\sigma^2\mathbf{G}$ for $\rho = -\frac{3}{24s+24qs+37}$.

Corollary 2.9. If the parameters of the ternary balanced block design are equal to $v = 4s+3$, $b_1 = 2(4s+3)$, $r_1 = 4(2s+1)$, $k_1 = 2(2s+1)$, $\lambda_1 = 8s+1$, $\rho_{11} = 2(2s+1)$, $\rho_{21} = 2s+1$ and $v = 4s+3$, $b_2 = 4(4s+q+2)$, $r_2 = 4(4s+q+2)$, $k_2 = 4s+3$, $\lambda_2 = 2(8s+2q+3)$, $\rho_{12} = 4(2s+q+1)$, $\rho_{22} = 2(2s+1)$, $s = 4,5,\ldots$, $q = 1,2,\ldots$, then $\mathbf{X} \in \mathbf{\Phi}_{2(12s+2q+7)\times(4s+3),\, 4(3s+2)}\{-1, 0, 1\}$ given by (2.1) is the regular D-optimal chemical balance weighing design with the covariance matrix of errors $\sigma^2\mathbf{G}$ for $\rho = -\dfrac{3}{72s+12q+43}$.

Corollary 2.10. If the parameters of the ternary balanced block design are equal to $v = 4s+3$, $b_1 = 4s+3$, $r_1 = 4s+3$, $k_1 = 4s+3$, $\lambda_1 = 2(2s+1)$, $\rho_{11} = 1$, $\rho_{21} = 2s+1$ and $v = 4s+3$, $b_2 = 2q(4s+3)$, $r_2 = 8q(s+1)$, $k_2 = 4(s+1)$, $\lambda_2 = 2q(4s+5)$, $\rho_{12} = 4q(2s+1)$, $\rho_{22} = 2q$, $s, q = 1,2,\ldots$, then $\mathbf{X} \in \mathbf{\Phi}_{(2q+1)(4s+3)\times(4s+3),2(2s+q+1)}\{-1, 0, 1\}$ given by (2.1) is the regular D-optimal chemical balance weighing design with the covariance matrix of errors $\sigma^2\mathbf{G}$ for $\rho = -\dfrac{1}{2(2s+4qs+2q^2+3q+1)}$.

3. Discussion

In the paper Ceranka and Graczyk (2014b), the construction of the regular D-optimal chemical balance weighing design is also given. This construction is implemented on the base of the incidence matrices of the balanced bipartite weighing designs. For any class $\mathbf{\Phi}_{n\times p,m}\{-1, 0, 1\}$ and for any ρ in (1.1), we are not able to construct the design matrix of the regular D-optimal chemical balance weighing design. In Ceranka and Graczyk (2014b), we determine D-optimal chemical balance weighing design, among others, for

$$\rho \in \left\{-\frac{1}{13}, -\frac{1}{21}, -\frac{1}{29}, -\frac{2}{83}, -\frac{1}{56}, -\frac{1}{67}\right\}.$$

Whereas, in present paper we give the construction including cases $\rho \in \left\{-\frac{1}{9}, -\frac{1}{13}, -\frac{1}{17}, -\frac{1}{23}, -\frac{1}{26}, -\frac{1}{29}\right\}$. So it seems to be necessary to prospect new construction methods determining the regular D-optimal chemical balance weighing design for other values of ρ and in different classes $\mathbf{\Phi}_{n\times p,m}\{-1,0,1\}$.

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