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COMPARISON OF TWO METHODS OF TESTING HYPOTHESES ON NATURAL OCCURRENCE OF PESTS IN FIELD CROPS

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Summary

This article introduces a new view on testing hypotheses for a special class of experiments. For certain null experiments, it is shown that for testing several hypotheses it is appropriate to implement the nested test procedure. However, if certain conditions hold, a separate test procedure for testing the joint hypothesis is applied. A comparison is made between these methods of testing for hypotheses on natural pest occurrence in a field crop.

Keywords and phrases: nested test procedure, separate test procedure, null experiment, pest in field crop

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1. Introduction

Testing hypotheses is an essential part of statistical inference. In a case where there are joint hypotheses to be tested, we decide whether to test jointly using the nested test procedure or jointly using the separate test procedure. This problem is described for example by Scheffe (1959), Seber (1980) and Caliński (2005). These authors consider the use of the nested test procedure for the same statistical problem, to estimate the degree of polynomial regression. Seber wrote: "the nested procedure can be applied to a set of hypotheses in which there is a natural ordering of the hypotheses".

In this paper we describe a special research problem relating to issues of plant protection. Investigation of the effectiveness of plant protection products, such as insecticides, acaricides and molluscicides, should be preceded by monitoring of the experimental field . The estimation of the pest's population and its dispersion on the experimental field are important aspects of the planning of the experiment. For this reason we may carry out a null experiment. The null experiment is important because it confirms, or not, that the assumed model is correct.

2. Joint tests

If we have $v>2$ alternative treatments to be compared, we group the experimental units into sets, for example v, the units in each set being expected to give as nearly as possible the same observation if these units were untreated. Each set is called a block, and the sequence of all the sets is called a system of blocks. We can implement s≥1 systems of blocks to increase the precision of the investigation and to control the remaining variation (Cox, 1958).

Let **v** be the n-dimensional vector of observations. A linear additive fixed effects model of the form

$$
y = X\xi + \varepsilon \tag{2.1}
$$

is assumed for the observations, where **X** is the appropriate n x m matrix of rank no greater then m, $ξ$ denotes the m dimensional vector of parameters, and $ε$ is the vector of uncorrelated random variables each having normal distributions with expectation zero and variance σ^2 ($\varepsilon \sim N(0;\sigma^2I)$). This is the model of variance analysis (Rao, 1982; Hinkelmann and Kempthorne,1994).

We consider a sequence of linear models

$$
G: \mathbf{y} \sim N(\mathbf{X}\boldsymbol{\xi}; \sigma^2 \mathbf{I}),
$$

\n
$$
H_1: \mathbf{y} \sim N(\mathbf{X}_1 \boldsymbol{\xi}_1; \sigma^2 \mathbf{I}),
$$

\n
$$
H_{12}: \mathbf{y} \sim N(\mathbf{X}_{12} \boldsymbol{\xi}_{12}; \sigma^2 \mathbf{I}),
$$

\n
$$
H_{123}: \mathbf{y} \sim N(\mathbf{X}_{123} \boldsymbol{\xi}_{123}; \sigma^2 \mathbf{I}),
$$

\n(2.2)

where $R(X_{123})$ ⊂ $R(X_{12})$ ⊂ $R(X_1)$ ⊂ $R(X)$ ⊂ $R(X^n)$, where $R(X)$ denotes the linear subspace spanned by the columns of the matrix **X**. Our main purpose is to test the hypothesis H_{123} against G. However, in order to obtain more information about the reduced model, we will test the hypotheses: H_1 against G, H_{12} against H_1 and H_{123} against H_{12} .

Let us define the projectors

$$
\mathbf{P}_{123} = \mathbf{X}_{123} (\mathbf{X}_{123}' \mathbf{X}_{123})^{-} \mathbf{X}_{123}' , \ \mathbf{P}_{12} = \mathbf{X}_{12} (\mathbf{X}_{12}' \mathbf{X}_{12})^{-} \mathbf{X}_{12}', \tag{2.3}
$$
\n
$$
\mathbf{P}_{1} = \mathbf{X}_{1} (\mathbf{X}_{1}' \mathbf{X}_{1})^{-} \mathbf{X}_{1}' , \ \mathbf{P}_{0} = \mathbf{X} (\mathbf{X}' \mathbf{X})^{-} \mathbf{X}',
$$

where $P_0P_1=P_1$, $P_0P_{12}=P_{12}$, $P_0P_{123}=P_{123}$, $P_1P_{12}=P_{12}$, $P_1P_{123}=P_{123}$ and $P_{12}P_{123}=P_{123}$ (Rao, Mitra, 1980) and $(\mathbf{X}'\mathbf{X})$ ⁻ denotes the generalized inverse of the matrix $(X'X)$. Now we can take the F statistics for testing the sequence hypotheses H_1 , $H₁₂, H₁₂₃$, being in the form

$$
F_1 = \frac{n - r(X)}{r(X) - r(X_1)} \frac{(X\hat{\xi} - X_1\hat{\xi}_1)'(X\hat{\xi} - X_1\hat{\xi}_1)}{(y - X\hat{\xi})'(y - X\hat{\xi})},
$$

\n
$$
F_{12} = \frac{n - r(X_1)}{r(X_1) - r(X_{12})} \frac{(X_1\hat{\xi}_1 - X_{12}\hat{\xi}_{12})'(X_1\hat{\xi}_1 - X_{12}\hat{\xi}_{12})}{(y - X_1\hat{\xi}_1)'(y - X_1\hat{\xi}_1)},
$$
(2.4)
\n
$$
F_{123} = \frac{n - r(X_{12})}{r(X_{12}) - r(X_{123})} \frac{(X_{12}\hat{\xi}_{12} - X_{123}\hat{\xi}_{123})'(X_{12}\hat{\xi}_{12} - X_{123}\hat{\xi}_{123})}{(y - X_{12}\hat{\xi}_{12})'(y - X_{12}\hat{\xi}_{12})},
$$

where $\hat{\xi}$ denotes the least-squares estimator of the vector ξ , r(.) denotes the rank of the matrix (.). It is known that these statistics have independent F distributions and the hypothesis H_{123} is accepted against G if the each nested hypothesis H_1 , H_{12} , H_{123} is accepted (Seber, 1980; Caliński, 2005).

 When the order of nesting hypotheses is not natural, there is the option of testing the hypotheses directly in separate tests. Let us denote

$$
H_i: \mathbf{y} \sim N(\mathbf{X}_i \boldsymbol{\xi}_i; \sigma^2 \mathbf{I}), i=1,2,3,
$$
 (2.5)

where $R(X_i) \subset R(X)$. Now for projectors $P_i = X_i (X_i' X_i)^T X_i'$, we assume that the equation

$$
(\mathbf{P}_0 - \mathbf{P}_i)(\mathbf{P}_0 - \mathbf{P}_i) = \mathbf{0} \text{ for } i, i' = 1, 2, 3, i \neq i'
$$
 (2.6)

holds. Testing of H_i against G can be done using a statistic of the form

$$
F_i = \frac{n - r(\mathbf{X})}{r(\mathbf{X}) - r(\mathbf{X}_i)} \frac{(\mathbf{X}\hat{\boldsymbol{\xi}} - \mathbf{X}_i\hat{\boldsymbol{\xi}}_i)'(\mathbf{X}\hat{\boldsymbol{\xi}} - \mathbf{X}_i\hat{\boldsymbol{\xi}}_i)}{(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\xi}})'(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\xi}})} \text{ for } i=1,2,3. \tag{2.7}
$$

In this approach, the hypothesis H_{123} is accepted against G when each hypothesis H_i is separately accepted.

3. Natural occurrence of a pest in a field crop

In 2004 research was done into the occurrence and harmfulness of *Arianta arbustorum* on a plantation of spring rape established after many years as a meadow. In the germination period the location of the pest on the plantation and the level of plant damage were investigated. The mean percentage of damaged rape seedlings on the plantation was 13.5%. *A. arbustorum* occurs in damp environments, mainly in scrubs next to rivers, meadows, forests, parks and gardens. It occurs in especially large amounts in places situated near to irrigation canals. The plantation of the spring rape was adjacent to roads and near to irrigation canals.

In the planning main experiment the treatments mean the five levels of application of a certain plant protection product (2% methiocarb). The main purpose of the research was to investigate the influence of these treatments on damage to the rape seedlings caused by *A. arbustorum*.

Before the main experiment, the null experiment was carried out. The purpose of the investigation was to verify the occurrence of the pest on the experimental units. The numbers of the pest population on two small plots of 1 $m²$ area in each experimental unit on the spring rape plantation were observed. The observations (analyzed data) were averages of the numbers of pests from two plots in each unit. The null experiment concerning the natural occurrence of *A. arbustorum* was carried out by the implementation of two systems of blocks. Twenty-five experimental units were grouped into five blocks, five units in each block. The second system of blocks was used simultaneously. In general, the first system of blocks we shall call rows, and the second one columns. The first column consists of units 1, 6, 11, 16 and 21, the second of units 2, 7, 12, 17 and 22, and so on (Fig. 1). When each treatment occurs once in each row and once in each column we obtain a Latin square arrangement. The average number of individuals per 1 $m²$ area was obtained for each unit.

Fig. **1**. Scheme of occurrence of treatments on experimental units

1,2,3,4,5 – number of treatment 1,2,…,25 – number of experimental units

In this case the model of observation (2.1) has the form

$$
\mathbf{y} = \left[\mathbf{1} \,|\, \mathbf{\Delta}' | \,\mathbf{D}'_1 | \,\mathbf{D}'_2\right] \cdot \begin{bmatrix} \mu \\ \boldsymbol{\alpha} \\ \boldsymbol{\beta} \\ \boldsymbol{\gamma} \end{bmatrix} + \boldsymbol{\epsilon},\tag{3.1}
$$

where **1** is the n-dimensional vector of ones, ∆**'** denotes the (n x v)-dimensional design matrix for treatments, \mathbf{D}'_1 denotes the (n x v)-dimensional design matrix for rows, \mathbf{D}'_2 denotes the (n x v)-dimensional design matrix for columns, μ is an overall mean parameter, α is the v-dimensional vector of treatments effects, $β$ is the v-dimensional vector of rows effects, $γ$ is the v-dimensional vector of columns effects and ε is the n-dimensional vector of uncorrelated random variables each having expectation zero and variance σ^2 . The model (3.1) describes a Latin square design, where v distinct treatments are allocated to experimental units arranged in v rows and v columns. The allocation can be described by the incidence matrices $N_1 = \Delta D'_1 = 11'$, $N_2 = \Delta D'_2 = 11'$ and $N_3 = D_1 D'_2 = 11'$. The vector of treatment replications is $\mathbf{r} = \mathbf{N}_1 \mathbf{1} = \mathbf{N}_2 \mathbf{1} = \mathbf{V} \mathbf{1}$. We assume that **y** has the n-dimensional normal distribution.

We are interested in testing the hypothesis H_{123} : $y \sim N(\mu 1; \sigma^2 I)$ against G. The nested procedure can be applied, because there is a natural ordering of the

elimination sets of parameters. We will verify the hypotheses (2.2). We can write the hypotheses in the following forms:

$$
H_1: \mathbf{y} \sim N([\mathbf{1}|\Delta'|\mathbf{D}'_1] \cdot \begin{bmatrix} \mu \\ \alpha \\ \beta \end{bmatrix}; \sigma^2 \mathbf{I}),
$$

\n
$$
H_{12}: \mathbf{y} \sim N([\mathbf{1}|\Delta'] \cdot \begin{bmatrix} \mu \\ \alpha \end{bmatrix}; \sigma^2 \mathbf{I}),
$$

\n
$$
H_{123}: \mathbf{y} \sim N(\mu \mathbf{1}; \sigma^2 \mathbf{I}).
$$
\n(3.2)

We can write the hypotheses (3.2) in the different forms

$$
H_1: \gamma = 0, H_{12}: [\boldsymbol{\beta}' | \gamma']' = 0, H_{123}: [\boldsymbol{\alpha}' | \boldsymbol{\beta}' | \gamma']' = 0.
$$
 (3.3)

We observe that $r([\Delta' | D'_1 | D'_2])=3v-2=13$, $r([\Delta' | D'_1])=2v-1=9$, $r(\Delta')=v=5$. Hence we have $R(1) \subset R(\Delta') \subset R([\Delta' | D'_1]) \subset R([\Delta' | D'_1 | D'_2]) \subset R(\mathbb{R}^{25})$. The projectors (2.3) have the forms

$$
\mathbf{P}_{123} = \frac{1}{\mathbf{v}^2} \mathbf{11}', \ \mathbf{P}_{12} = \boldsymbol{\Delta}' (\boldsymbol{\Delta} \boldsymbol{\Delta}')^{-1} \boldsymbol{\Delta} = \frac{1}{\mathbf{v}} \boldsymbol{\Delta}' \boldsymbol{\Delta},
$$
\n
$$
\mathbf{P}_1 = [\boldsymbol{\Delta}' | \mathbf{D}'_1] (\begin{bmatrix} \boldsymbol{\Delta} \\ \mathbf{D}_1 \end{bmatrix} [\boldsymbol{\Delta}' | \mathbf{D}'_1])^{-1} (\begin{bmatrix} \boldsymbol{\Delta} \\ \mathbf{D}_1 \end{bmatrix}),
$$
\n
$$
\mathbf{P}_0 = [\boldsymbol{\Delta}' | \mathbf{D}'_1 | \mathbf{D}'_2] (\begin{bmatrix} \boldsymbol{\Delta} \\ \mathbf{D}_1 \\ \mathbf{D}_2 \end{bmatrix} [\boldsymbol{\Delta}' | \mathbf{D}'_1 | \mathbf{D}'_2])^{-1} (\begin{bmatrix} \boldsymbol{\Delta} \\ \mathbf{D}_1 \\ \mathbf{D}_2 \end{bmatrix}.
$$

On the basis of measurements we calculate means and standard deviations for the observed variables (Table 1).

Column		mean	st. dv.	min	median	max
1	5	48.5	21.3	24.5	44.0	83.0
2	5	55.8	35.6	24.5	47.5	117.0
3	5	55.7	31.2	20.0	70.5	83.0
$\overline{4}$	5	57.0	20.6	40.4	43.5	85.0
5	5	62.6	18.6	44.0	60.0	91.5
Row		mean	st. dv.	min	median	max
1	5	55.7	18.9	40.5	44.0	83.0
2	5	52.4	39.4	20.0	40.5	117.0
3	5	54.2	22.9	24.5	49.5	81.0
$\overline{4}$	5	51.3	24.7	24.0	48.0	91.5
5	5	66.0	19.3	44.0	70.5	85.0
Treatment		mean	st. dv.	min	median	max
1	5	35.4	10.3	24.0	40.5	44.0
2	5	75.2	29.7	43.5	83.0	117.0
3	5	58.4	14.1	40.5	60.0	73.0
$\overline{4}$	5	61.6	24.9	24.5	68.0	85.0
5	5	49.0	26.1	20.0	43.0	91.5

Table 1. Means of numbers of *A. arbustorum* per 1 m² area of experimental unit grouped into two systems of blocks

We assume the significance level α =0.05 and we verify each nested hypothesis H_1 , H_{12} , H_{123} at the same significance level

$$
\alpha_n = 1 - (1 - \alpha)^{\frac{1}{3}} = 0.01695 \approx 0.017. \tag{3.4}
$$

Hence we obtain values of the statistics (2.4)

$$
F_1 = \frac{25 - 13}{13 - 9} \cdot \frac{504.54}{8760.6} = 0.173,
$$

$$
F_{12} = \frac{25 - 9}{9 - 5} \cdot \frac{691.74}{9265.2} = 0.299,
$$

$$
F_{123} = \frac{25 - 5}{5 - 1} \cdot \frac{4395.4}{9956.9} = 2.207.
$$

For these statistics p-values are calculated:

$$
p(0.173;4;12) = 0.948 > 0.017,
$$

\n
$$
p(0.299;4;16) = 0.874 > 0.017,
$$

\n
$$
p(2.207;4;20) = 0.105 > 0.017,
$$
\n(3.5)

where 0.017 is the value of the significance level (3.4). In this case we accept all hypotheses (3.3) and thus we accept the joint hypothesis too.

On the other hand we can verify the hypothesis H_{123} : $y \sim N(\mu 1; \sigma^2 I)$ against G using the separate test procedure. Now we observe that the orthogonal condition (2.6) takes the following form:

$$
\Delta(I_{25} - P_2)(I_{25} - P_1)D'_1 = \Delta(I_{25} - P_3)(I_{25} - P_1)D'_2
$$

$$
= {\bf D}'_1({\bf I}_{25}-{\bf P}_3)({\bf I}_{25}-{\bf P}_2){\bf D}'_2=0,
$$

where

$$
\mathbf{P}_2 = [\mathbf{\Delta}' \mid \mathbf{D}'_2] \begin{bmatrix} \mathbf{\Delta} \\ \mathbf{D}_2 \end{bmatrix} [\mathbf{\Delta}' \mid \mathbf{D}'_2] \begin{bmatrix} \mathbf{\Delta} \\ \mathbf{D}_2 \end{bmatrix},
$$

$$
\mathbf{P}_3 = [\mathbf{D}'_1 \mid \mathbf{D}'_2] \begin{bmatrix} \mathbf{D}_1 \\ \mathbf{D}_2 \end{bmatrix} [\mathbf{D}'_1 \mid \mathbf{D}'_2] \begin{bmatrix} \mathbf{D}_1 \\ \mathbf{D}_2 \end{bmatrix}.
$$

We can see the truth of the above condition when we write down it in terms of the incidence matrices. The orthogonal condition can be written as

$$
\frac{1}{v^2}N_1N_1'N_1 = \frac{1}{v^2}N_2N_2'N_2 = \frac{1}{v^2}N_3N_3'N_3 = N_1 = N_2 = N_3 = 11'.
$$

We can write the separate hypotheses (2.5) in the following forms

$$
H_1: \mathbf{y} \sim N([\mathbf{1} \mid \boldsymbol{\Delta}' \mid \mathbf{D}'_1] \cdot \begin{bmatrix} \boldsymbol{\mu} \\ \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{bmatrix}; \sigma^2 \mathbf{I}),
$$

\n
$$
H_2: \mathbf{y} \sim N([\mathbf{1} \mid \boldsymbol{\Delta}' \mid \mathbf{D}'_2] \cdot \begin{bmatrix} \boldsymbol{\mu} \\ \boldsymbol{\alpha} \\ \boldsymbol{\gamma} \end{bmatrix}; \sigma^2 \mathbf{I}),
$$
(3.6)
\n
$$
H_3: \mathbf{y} \sim N([\mathbf{1} \mid \mathbf{D}'_1 \mid \mathbf{D}'_2] \cdot \begin{bmatrix} \boldsymbol{\mu} \\ \boldsymbol{\beta} \\ \boldsymbol{\gamma} \end{bmatrix}; \sigma^2 \mathbf{I}).
$$

The hypotheses (3.6) can be written in the following different forms

$$
H_1: \gamma = 0, H_2: \beta = 0, H_3: \alpha = 0.
$$
 (3.7)

We assume again the significance level α =0.05 and we verify each separate hypothesis H_1 , H_2 , H_3 at the same significance level. In this case we have

$$
\alpha_s = \frac{\alpha}{3} = 0.01667 \approx 0.017. \tag{3.8}
$$

We obtain the values of the statistics (2.7)

$$
F_1 = \frac{25 - 13}{13 - 9} \cdot \frac{504.54}{8760.6} = 0.173,
$$

$$
F_2 = \frac{25 - 13}{9 - 5} \cdot \frac{691.74}{8760.6} = 0.237,
$$

$$
F_1 = \frac{25 - 13}{9 - 5} \cdot \frac{4395.4}{8760.6} = 0.237,
$$

$$
F_3 = \frac{25 - 13}{5 - 1} \cdot \frac{4395.4}{8760.6} = 1.500
$$

and p-values for the statistics

$$
p(0.173;4;12) = 0.948 > 0.017,
$$

\n
$$
p(0.237;4;12) = 0.912 > 0.017,
$$

\n
$$
p(1.5;4;12) = 0.263 > 0.017,
$$
\n(3.9)

where 0.017 is the value of the significance level (3.8). Testing the joint hypothesis H_{123} : $y \sim N(\mu 1; \sigma^2 I)$ against G, using the separate test procedure, in this case we accept each separate hypothesis (3.7) and the joint hypothesis too.

4. Conclusion

In the considered research problem we accepted the joint hypothesis using both the nested procedure as well as the separate procedure. We affirmed the homogeneity of the dispersion of *A. arbustorum* on the experimental field . For each experimental unit the pest population is at the same level. Hence the main experiment should be planned using systems of blocks dependent on the variation of the experimental field environment (the moisture of the soil, the richness of the soil in mineral elements, and so on).

We observed that the nominators of the statistics F_1 , F_{12} , F_{123} in the formulas (2.4), and the nominators of the statistics F_1 , F_2 , F_3 in the formulas (2.7), are respectively the same. The denominators of these statistics are different. We have a greater probability of rejecting the second and third nested hypotheses H_{12} and H_{123} than the separate hypotheses H_2 and H_3 respectively. There are differences between the p-values (3.5) and (3.9). These differences are small, but for a particular research problem they may be important. We claim that there is a natural ordering of the hypotheses, so we implement the nested test procedure than the separate test procedure, subject to the necessity of the truth of the orthogonal condition.

The planning experiments in plant protection research should include actual monitoring of the growth in numbers of pest populations. This type of planning may be more effective, and the results of the investigation will be obtained with greater precision.

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PORÓWNANIE DWÓCH METOD TESTOWANIA HIPOTEZ DLA ROZPRZESTRZENIENIA SZKODNIKA W UPRAWIE POLOWEJ

Streszczenie

W pracy przedstawiono pewne podejście do problemu testowania hipotez w szczególnej klasie eksperymentów jakimi są eksperymenty zerowe. Eksperymenty zerowe przeprowadza się w wielu zagadnieniach z ochrony roślin, są to wstępne badania mające na celu określenie równomierności rozprzestrzenienia szkodnika na polu doświadczalnym. Odpowiedni schemat rozmieszczenia etykiet obiektów na pogrupowanych jednostkach doświadczalnych w s systemach blokowych zapewnia możliwość zastosowania zagnieżdżonej procedury testowej. Przy spełnieniu warunków ortogonalności możliwe jest zastosowanie procedury oddzielnych testów. Te dwie procedury są porównane dla badania rozprzestrzenienia *A. arbustorum* na plantacji rzepaku jarego.

Słowa kluczowe: procedura testów zagnieżdżonych, procedura oddzielnych testów, zerowy eksperyment, szkodnik uprawy polowej

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