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SOME NOTES ABOUT CHEMICAL BALANCE WEIGHING DESIGN FOR $p = v + 1$ OBJECTS BASED ON BALANCED BLOCK DESIGNS

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Summary

The problem of estimation of individual (weights) measurements of p objects using n measurements operations according to the model of the chemical balance weighing design is presented. Assuming that not all objects in each measurement are included, the optimality conditions and the construction methods of the design matrix **X** of the optimum chemical balance weighing design for p = v + 1 objects are given. The construction is based on the incidence matrices of the balanced incomplete block designs and the balanced bipartite weighing designs for v treatments.

Key words and phrases: balanced bipartite weighing design, balanced incomplete block design, chemical balance weighing design

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1. Introduction

We consider an experiment in which using n measurement (weighing) operations we determine unknown weights of given number of $p = v + 1$ objects. The results of the experiment can be described by the model

$$\mathbf{y} = \mathbf{X}\mathbf{w} + \mathbf{e} \tag{1.1}$$

where $\mathbf{y}$ is $n \times 1$ random vector of the observed weights, $\mathbf{X} = (x_{ij})$, $x_{ij} = -1, 0, 1$, $i = 1,2,...,n$, $j = 1,2,..., p$, is called a design matrix, $\mathbf{w}$ is $p \times 1$ vector representing unknown weights of objects. We assume that in the model (1.1) errors are not systematic, have different variances and they are uncorrelated, i.e. $\mathrm{E}(\mathbf{e}) = \mathbf{0}_n$ and $\mathrm{E}(\mathbf{e}\mathbf{e}') = \sigma^2 \mathbf{G}$, where $\mathbf{0}_n$ is an $n \times 1$ null vector, $\mathbf{G}$ is $n \times n$ positive definite diagonal matrix of known elements and $\mathbf{e}'$ denotes transposition of $\mathbf{e}$. To estimate unknown weights of objects we use the least squares method giving the following formula

$$\hat{\mathbf{w}} = \left(\mathbf{X}'\mathbf{G}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{G}^{-1}\mathbf{y}$$

with the dispersion matrix of $\hat{\mathbf{w}}$ as

$$\mathrm{V}(\hat{\mathbf{w}}) = \sigma^2 \left(\mathbf{X}'\mathbf{G}^{-1}\mathbf{X}\right)^{-1}, \tag{1.2}$$

if $\mathbf{X}$ is of full column rank, i.e. $\mathrm{r}(\mathbf{X}) = p$.

Several problems connected of the optimality in the chemical balance weighing designs have been studied in Raghavarao (1971) and Shah and Sinha (1989). It is well known that the minimum attainable variance for each of the estimated weights for a chemical balance weighing design is equal to σ^2/n and each variance of the estimator of unknown weights of objects attains the minimum if and only if $\mathbf{X}'\mathbf{X} = n\mathbf{I}_p$. This design is called optimum chemical balance weighing design. In the optimal design the elements of $\mathbf{X}$ are -1 and 1, only. In Banerjee (1975) the construction methods of such design matrices are available.

In the paper we assume that not all objects in each measurement operation are included, i.e. some elements of $\mathbf{X}$ can be null. We present new construction method of the optimum design matrix for $p = v + 1$ objects based on the incidence matrices of the balanced incomplete block designs and the balanced bipartite weighing designs for v treatments. Ceranka and Graczyk (2001a,b; 2004) gave optimality conditions and construction methods of such design matrices but for other numbers of objects and for other dispersion matrices.

2. Variance limit of estimated weights

Each form of the dispersion matrix (1.2) requires of separate investigation. Let us consider the matrix $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is given as

$$\mathbf{G} = \begin{bmatrix} \frac{1}{a}\mathbf{I}_{n_1} & \mathbf{0}_{n_1}\mathbf{0}'_{n_2} & \mathbf{0}_{n_1}\mathbf{0}'_{n_2} \\ \mathbf{0}_{n_2}\mathbf{0}'_{n_1} & \mathbf{I}_{n_2} & \mathbf{0}_{n_2}\mathbf{0}'_{n_2} \\ \mathbf{0}_{n_2}\mathbf{0}'_{n_1} & \mathbf{0}_{n_2}\mathbf{0}'_{n_2} & \mathbf{I}_{n_2} \end{bmatrix}, \tag{2.1}$$

where $a > 0, \quad a \neq 1, \quad n = n_1 + n_2 + n_3$. Let $\mathbf{X}$ be partitioned as the matrix $\mathbf{G}$

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \mathbf{X}_3 \end{bmatrix}, \tag{2.2}$$

where m_{hj} is the number of times in which j th object is weighed for a particular matrix $\mathbf{X}_h = \left(x_{qj}^h\right)$, $q = 1,2,...,n_h$, $j = 1,2,..., p$, $h = 1,2,3.$ From Section 1e. 1 (ii) (b) given by Rao (1982) we have

Lemma 2.1. For the matrix $\mathbf{G}$ of the form (2.1), the matrix $\mathbf{X}$ of the rank p and a $p \times 1$ vector $\mathbf{c}$, the inequality

$$\mathbf{c}'\left(\mathbf{X}'\mathbf{G}^{-1}\mathbf{X}\right)^{-1}\mathbf{c} \geq \frac{\left(\mathbf{c}'\mathbf{c}\right)^2}{\mathbf{c}'\mathbf{X}'\mathbf{G}^{-1}\mathbf{X}\mathbf{c}} \tag{2.3}$$

holds with equality attained if and only if $\mathbf{c}$ is an eigenvector of $\mathbf{X}'\mathbf{G}^{-1}\mathbf{X}$.

Theorem 2.1. In the nonsingular chemical balance weighing design with the design matrix $\mathbf{X}$ given in (2.2) and the dispersion matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is of the form (2.1), $\mathrm{V}\left(\hat{w}_j\right) \geq \sigma^2/m$, where $m = am_1 + m_2 + m_3$, $\quad m_h = \max\{m_{h1}, m_{h2}, ..., m_{hp}\}$, $h = 1,2,3$.

Proof. Let $\mathbf{c}_j$, $j = 1,2,..., p,$ be the j th column of $\mathbf{I}_p$. Then $\hat{w}_j = \mathbf{c}_j^{'}\hat{\mathbf{w}}$ and $\mathrm{V}(\hat{w}_j) = \sigma^2 \mathbf{c}_j^{'}(\mathbf{X}^{'}\mathbf{G}^{-1}\mathbf{X})^{-1}\mathbf{c}_j$. Since $\mathrm{r}(\mathbf{X}) = p$, then from Lemma 2.1 we have

$$\mathrm{V}(\hat{w}_j) \geq \sigma^2 \frac{(\mathbf{c}_j^{'}\mathbf{c}_j)}{\mathbf{c}_j^{'}\mathbf{X}^{'}\mathbf{G}^{-1}\mathbf{X}\mathbf{c}_j} = \tag{2.4}$$

$$\frac{\sigma^2}{a\sum_{q=1}^{n_1}(x_{qj}^1)^2 + \sum_{q=1}^{n_2}(x_{qj}^2)^2 + \sum_{q=1}^{n_3}(x_{qj}^3)^2} \geq \frac{\sigma^2}{am_1 + m_2 + m_3} = \frac{\sigma^2}{m},$$

because elements $x_{qj}^h = -1, 0, 1,$ only, and the number of elements equal –1 or 1 in each column of $\mathbf{X}_h$ is m_{hj}. Hence the theorem is proved.

Definition 2.1. Any chemical balance weighing design with the design matrix $\mathbf{X}$ given in (2.2) and the dispersion matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is of the form (2.1), is called optimal for the estimation of individual weights if $\mathrm{V}(\hat{w}_j) = \sigma^2/m$ for all j, $j = 1,2,...,p$.

Theorem 2.2. Any nonsingular chemical balance weighing design with the design matrix $\mathbf{X}$ given in (2.2) and the dispersion matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is of the form (2.1), is optimal if and only if

$$\mathbf{X}^{'}\mathbf{G}^{-1}\mathbf{X} = m\mathbf{I}_p. \tag{2.5}$$

Proof. To prove the necessity condition we observe that the equality in (2.4) holds for any $j = 1,2,..., p$ if and only if $\mathbf{X}^{'}\mathbf{G}^{-1}\mathbf{X}\mathbf{c}_j = \mu_j\mathbf{c}_j$, $\mu_j > 0$ and $\mathbf{c}_j^{'}\mathbf{X}^{'}\mathbf{G}^{-1}\mathbf{X}\mathbf{c}_j = am_1 + m_2 + m_3$. These conditions imply that $\mathbf{X}^{'}\mathbf{G}^{-1}\mathbf{X} = \mathrm{diag}\{\mu_1, \mu_2, ..., \mu_p\}$ and $\mu_1 = \mu_2 = ... = \mu_p = am_1 + m_2 + m_3$. Then we have the condition (2.5). The proof for the sufficiency condition is obvious.

3. Balanced block design

Let us recall definitions of the balanced incomplete block design given in Raghavarao (1971) and the balanced bipartite weighing design given in Swamy (1982).

A balanced incomplete block design (BIBD) there is an arrangement of v treatments into b blocks, each of size k, in such a way, that each treatment occurs at most ones in each block, occurs in r blocks and every pair of treatments occurs together in λ blocks. The integers v, b, r, k, λ are called the parameters of BIBD. It is straightforward to verify that

$$vr = bk, \quad \lambda(v-1) = r(k-1), \quad \mathbf{NN}' = (r-\lambda)\mathbf{I}_v + \lambda \mathbf{1}_v\mathbf{1}_v',$$

where $\mathbf{N}$ denotes the incidence matrix of BIBD.

A balanced bipartite weighing design (BBWD) there is an arrangement of v treatments into b blocks in such a way that each block containing k distinct treatments is divided into 2 subblocks containing k_1 and k_2 treatments, respectively, where $k = k_1 + k_2$. Each treatment appears in r blocks. Each pair of treatments from different subblocks appears together in λ_1 blocks and each pair of treatments from the same subblock appears together in λ_2 blocks. The integers v, b, r, k_1, k_2, λ_1, λ_2 are called the parameters of the BBWD. Let $\mathbf{N}^*$ be the incidence matrix of such BBWD. The parameters of BBWD are dependent and fulfil the following equalities

$$vr = bk, \quad b = \frac{\lambda_1 v(v-1)}{2k_1k_2}, \quad \lambda_2 = \frac{\lambda_1[k_1(k_1-1)+k_2(k_2-1)]}{2k_1k_2},$$

$$r = \frac{\lambda_1 k(v-1)}{2k_1k_2}, \quad \mathbf{N}^*\mathbf{N}^{*'} = (r-\lambda_1-\lambda_2)\mathbf{I}_v + (\lambda_1+\lambda_2)\mathbf{1}_v\mathbf{1}_v'.$$

4. Construction method

It is very difficult, or sometimes not possible, to construct the optimum chemical balance weighing design for any number of objects and any number of measurement operations. In the present paper, we construct the design matrix of the optimum chemical balance weighing design for $p = v + 1$ objects based on the optimum chemical balance weighing design for $p = v$ objects. In Ceranka and Graczyk (2004) the construction methods of the optimum chemical balance weighing design for $p = v$ objects based on the same set of the incidence matrices of BIBD and BBWD are presented.

Let $\mathbf{N}_1$ be the incidence matrix of BIBD with the parameters $v,\ b_1,\ r_1,\ k_1,\ \lambda_1$, and $\mathbf{N}_2^*$ be the incidence matrix of BBWD with the parameters $v,\ b_2,\ r_2,\ k_{12},\ k_{22},\ \lambda_{12},\ \lambda_{22}\ (k_{12} < k_{22})$. Let us define $\mathbf{N}_2$ obtaining from $\mathbf{N}_2^*$ by replacing k_{12} elements equal +1 in each column which correspond to the elements belonging to the first subblock by –1. Thus each column of $\mathbf{N}_2$ will contain k_{12} elements equal to –1, k_{22} elements equal to 1 and $v - k_{12} - k_{22}$ elements equal to 0. Then, the design matrix $\mathbf{X}$ of the chemical balance weighing design has the form

$$\mathbf{X} = \begin{bmatrix} 2\mathbf{N}_1' - \mathbf{1}_{b_1}\mathbf{1}_v' & \mathbf{0}_{b_1} \\ \mathbf{N}_2' & \mathbf{1}_{b_2} \\ -\mathbf{N}_2' & \mathbf{1}_{b_2} \end{bmatrix}. \tag{4.1}$$

In this design $p = v + 1$, $n_1 = b_1, n_2 = n_3 = b_2$. It is easy to show that chemical balance weighing design $\mathbf{X}$ in (4.1) is nonsingular if and only if $v \neq 2k_1$ or $k_{12} \neq k_{22}$.

Theorem 4.1. Any nonsingular chemical balance weighing design with the design matrix $\mathbf{X}$ given in (4.1) and matrix $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is given in (2.1) is optimal if and only if

(i) $$ab_1 - 4a(r_1 - \lambda_1) + 2(\lambda_{22} - \lambda_{12}) = 0$$

and

(ii) $$ab_1 = 2(b_2 - r_2).$$

Proof. For the design matrix $\mathbf{X}$ given in (4.1) and $\mathbf{G}$ in (2.1) we have

$$\mathbf{X}'\mathbf{G}^{-1}\mathbf{X} = \begin{bmatrix} (4a(r_1-\lambda_1)+2(r_2-\lambda_{22}+\lambda_{12}))\mathbf{I}_v + (ab_1 - 4a(r_1-\lambda_1)+2(\lambda_{22}-\lambda_{12}))\mathbf{1}_v\mathbf{1}_v' & \mathbf{0}_v \\ \mathbf{0}_v' & 2b_2 \end{bmatrix}.$$

The thesis of Theorem 4.1 is a result derived from Theorem 2.2. Note that the condition (i) of Theorem 4.1 is the same as optimality condition for the design with $p = v$ objects.

Based on the parameters of the BIBD given in Raghavarao (1971) and the parameters of BBWD given in Huang (1976) and Ceranka, Graczyk (2005) we can construct incidence matrices of such designs, respectively, and as the next step, the design matrix $\mathbf{X}$ of the optimum chemical balance weighing design.

Theorem 4.2. If for a given a the parameters of BIBD are equal to $v = s^2$, $b_1 = \frac{s(s^2-1)(s^2-4)}{6}$, $r_1 = \frac{(s-1)(s^2-1)(s^2-4)}{12}$, $k_1 = \frac{s(s-1)}{2}$, $\lambda_1 = \frac{(s-2)(s^2-1)(s^2-4)}{24}$
and the parameters of BBWD are equal to

(i) $a = 3$, $v = s^2$, $b_2 = \frac{s^3(s^2-1)}{4}$, $r_2 = s(s^2-1)$, $k_{12} = 1$, $k_{22} = 3$, $\lambda_{12} = \lambda_{22} = \frac{3s}{2}$, $s \geq 4$,

(ii) $a = \frac{4(s^2-9)}{3(s^2-4)}$, $v = s^2$, $b_2 = \frac{s^3(s^2-1)}{9}$, $r_2 = s(s^2-1)$, $k_{12} = 3$, $k_{22} = 6$, $\lambda_{12} = \lambda_{22} = 4s$, $s \geq 6$,

s is even integer, then $\mathbf{X}$ given in (4.1) is the design matrix of the optimum chemical balance weighing design with the dispersion matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is of the form (2.1).

Proof. It is easy to see that the parameters of BIBD and BBWD satisfy conditions given in the Theorem 4.1.

Theorem 4.3. If for a given a the parameters of BIBD are equal to $v = s^2$, $b_1 = s(s^2-1)(s^2-4)$, $r_1 = \frac{(s-1)(s^2-1)(s^2-4)}{2}$, $k_1 = \frac{s(s-1)}{2}$, $\lambda_1 = \frac{(s-2)(s^2-1)(s^2-4)}{4}$

and the parameters of BBWD are equal to

(i) $a = \frac{1}{6}$, $v = s^2$, $b_2 = \frac{s^3(s^2-1)}{12}$, $r_2 = \frac{s(s^2-1)}{3}$, $k_{12} = 1$, $k_{22} = 3$, $\lambda_{12} = \lambda_{22} = \frac{s}{2}$, $s \geq 4$,

(ii) $a = \frac{2(s^2-9)}{9(s^2-4)}$, $v = s^2$, $b_2 = \frac{s^3(s^2-1)}{9}$, $r_2 = s(s^2-1)$, $k_{12} = 3$, $k_{22} = 6$, $\lambda_{12} = \lambda_{22} = 4s$, $s \geq 6$,

s is even integer, then $\mathbf{X}$ given in (4.1) is the design matrix of the optimum chemical balance weighing design with the dispersion matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is of the form (2.1).

Theorem 4.4. If for a given a the parameters of BIBD are equal to $v = s^2$, $b_1 = s(s^2-1)(s^2-4)$, $r_1 = \frac{(s-1)(s^2-1)(s^2-4)}{2}$, $k_1 = \frac{s(s-1)}{2}$, $\lambda_1 = \frac{(s-2)(s^2-1)(s^2-4)}{4}$

and the parameters of BBWD are equal to

(i) $a = \frac{1}{3}$, $v = s^2$, $b_2 = \frac{s^3(s^2-1)}{6}$, $r_2 = \frac{2s(s^2-1)}{3}$, $k_{12} = 1$, $k_{22} = 3$, $\lambda_{12} = \lambda_{22} = s$, $s \geq 3$,

(ii) $a = \frac{s^2-9}{6(s^2-4)}$, $v = s^2$, $b_2 = \frac{s^3(s^2-1)}{12}$, $r_2 = \frac{3s(s^2-1)}{4}$, $k_{12} = 3$, $k_{22} = 6$, $\lambda_{12} = \lambda_{22} = 3s$, $s \geq 5$,

s is odd integer, then $\mathbf{X}$ given in (4.1) is the design matrix of the optimum chemical balance weighing design with the dispersion matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is of the form (2.1).

Theorem 4.5. If for a given a the parameters of BIBD are equal to $v = s^2$, $b_1 = \frac{2s(s^2-1)(s^2-9)}{9}$, $r_1 = \frac{(s-1)(s^2-1)(s^2-9)}{9}$, $k_1 = \frac{s(s-1)}{2}$, $\lambda_1 = \frac{(s-2)(s^2-1)(s^2-9)}{18}$

and the parameters of BBWD are equal to

(i) $a = \frac{3(s^2-4)}{4(s^2-9)}$, $v = s^2$, $b_2 = \frac{s^3(s^2-1)}{12}$, $r_2 = \frac{s(s^2-1)}{3}$,

$k_{12} = 1, k_{22} = 3, \lambda_{12} = \lambda_{22} = \frac{s}{2}$,

(ii) $a = \frac{s^2-4}{4(s^2-9)}$, $v = s^2$, $b_2 = \frac{s^3(s^2-1)}{4}$, $r_2 = s(s^2-1)$,

$k_{12} = 1, k_{22} = 3, \lambda_{12} = \lambda_{22} = \frac{3s}{2}$,

$s \geq 6$, s is even,

then $\mathbf{X}$ given in (4.1) is the design matrix of the optimum chemical balance weighing design with the dispersion matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is of the form (2.1).

Theorem 4.6. If for a given a the parameters of BIBD are equal to $v = s^2$, $b_1 = \frac{s(s^2-1)(s^2-4)}{3}$, $r_1 = \frac{(s-1)(s^2-1)(s^2-4)}{6}$, $k_1 = \frac{s(s-1)}{2}$, $\lambda_1 = \frac{(s-2)(s^2-1)(s^2-4)}{12}$

and the parameters of BBWD are equal to

(i) $a = 3$, $v = s^2$, $b_2 = \frac{s^3(s^2-1)}{2}$, $r_2 = 2s(s^2-1)$, $k_{12} = 1$,

$k_{22} = 3$, $\lambda_{12} = \lambda_{22} = 3s$,

(ii) $a = \frac{s^2-9}{2(s^2-4)}$, $v = s^2$, $b_2 = \frac{s^3(s^2-1)}{12}$, $r_2 = \frac{3s(s^2-1)}{4}$,

$k_{12} = 3, k_{22} = 6, \lambda_{12} = \lambda_{22} = 3s$,

$s \geq 5$, s is odd, except the case $s \equiv 3(\mathrm{mod}\, 6)$,

then $\mathbf{X}$ given in (4.1) is the design matrix of the optimum chemical balance weighing design with the dispersion matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is of the form (2.1).

Theorem 4.7. If for a given a the parameters of BIBD are equal to $v = s^2$, $b_1 = \frac{s(s^2-1)(s^2-9)}{3}$, $r_1 = \frac{(s-1)(s^2-1)(s^2-9)}{6}$, $k_1 = \frac{s(s-1)}{2}$, $\lambda_1 = \frac{(s-2)(s^2-1)(s^2-9)}{24}$

and the parameters of BBWD are equal to

(i) $a = \frac{s^2-4}{s^2-9}$, $v = s^2$, $b_2 = \frac{s^3(s^2-1)}{6}$, $r_2 = \frac{2s(s^2-1)}{3}$, $k_{12} = 1$, $k_{22} = 3$, $\lambda_{12} = \lambda_{22} = s$,

(ii) $a = \frac{3(s^2-4)}{s^2-9}$, $v = s^2$, $b_2 = \frac{s^3(s^2-1)}{2}$, $r_2 = 2s(s^2-1)$,

$k_{12} = 1, k_{22} = 3, \lambda_{12} = \lambda_{22} = 3s$,

$s \geq 5$, s is odd,

then $\mathbf{X}$ given in (4.1) is the design matrix of the optimum chemical balance weighing design with the dispersion matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is of the form (2.1).

Theorem 4.8. For $a = 2$, if the parameters of BIBD are equal to $v = 12,\ b_1 = 33,\ r_1 = 11,\ k_1 = 4,\ \lambda_1 = 3$ and the parameters of BBWD are equal to $v = 12,\ b_2 = 66,\ r_2 = 33,\ k_{12} = 2,\ k_{22} = 2,\ \lambda_{12} = 8,\ \lambda_{22} = 7$ then $\mathbf{X}$ given in (4.1) is the design matrix of the optimum chemical balance weighing design with the dispersion matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is of the form (2.1).

Theorem 4.9. For $a = \frac{1}{4}$, if the parameters of BIBD are equal to $v = 8,\ b_1 = 56,\ r_1 = 14,\ k_1 = 2,\ \lambda_1 = 2$ and the parameters of BBWD are equal to $v = 8,\ b_2 = 28,\ r_2 = 21,\ k_{12} = 2,\ k_{22} = 4,\ \lambda_{12} = 8,\ \lambda_{22} = 7$ then $\mathbf{X}$ given in (4.1) is the design matrix of the optimum chemical balance weighing design with the dispersion matrix of errors $\sigma^2\mathbf{G}$, where $\mathbf{G}$ is of the form (2.1).

References

Banerjee K.S. (1975). *Weighing designs for chemistry, medicine, economics, operations research, statistics*. Marcel Dekker Inc., New York.

Ceranka B., Graczyk M. (2001a). Chemical balance weighing designs under the restriction on the number of weighings. *Colloquium Biometryczne* 31, 39-45.

Ceranka B., Graczyk M. (2001b). Optimum chemical balance weighing designs under the restriction of weighings. *Discussiones Mathematicae – Probability and Statistics* 21, 111-120.

Ceranka B., Graczyk M. (2004). Optimum chemical balance weighting design with diagonal matrix of error based on balanced block designs. *Acta et Commentationes Universitatis Tartuensis de Mathematica* 8, 83 – 92.

Ceranka B., Graczyk M. (2005). About relations between the parameters of the balanced bipartite weighing designs. *Proceedings of the Fifth Workshop on Simulation*, S. M. Ermakov, V. B. Melas, A. N. Pepelyshev, Eds., Saint Petersburg University Publishers, 197-202.

Huang Ch. (1976). Balanced bipartite block designs. *Journal of Combinatorial Theory* (**A**) 21, 20-34.

Raghavarao D. (1971). *Constructions and Combinatorial Problems in designs of Experiments*. John Wiley Inc., New York.

Rao C.R. (1982). *Modele liniowe statystyki matematycznej*. PWN, Warszawa.Shah K.R. and Sinha B.K. (1989). *Theory of optimal designs*. Springer-Verlag, Berlin, Heidelberg.

Swamy M.N. (1982). Use of the balanced bipartite weighing designs. *Comm. Statist. Theory Methods* 11, 769-785.

OPTYMALNE CHEMICZNE UKŁADY WAGOWE DLA $p = v + 1$ OBIEKTÓW W OPARCIU O ZRÓWNOWAŻONE UKŁADY BLOKÓW

Streszczenie

W pracy przedstawiono estymację nieznanych miar $p = v + 1$ obiektów w n pomiarach w modelu chemicznego układu wagowego. Założono, że każdym pomiarze nie wszystkie obiekty biorą udział. Podano warunki, przy spełnieniu których istnienie chemicznego układu wagowego dla $p = v$ obiektów implikuje istnienie chemicznego układu wagowego dla $p = v + 1$ obiektów. Do konstrukcji macierzy układu optymalnego wykorzystano macierze incydencji układów zrównoważonych o blokach niekompletnych oraz dwudzielnych układów bloków.

Słowa kluczowe: chemiczny układ wagowy, dwudzielny układ bloków, układ zrównoważony o blokach niekompletnych

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